COS 484/584

L3: Text Classification

Spring 2021

Assignment 1 will be out later today — due Monday, Feb 22, 1:30pm (in 2 weeks)

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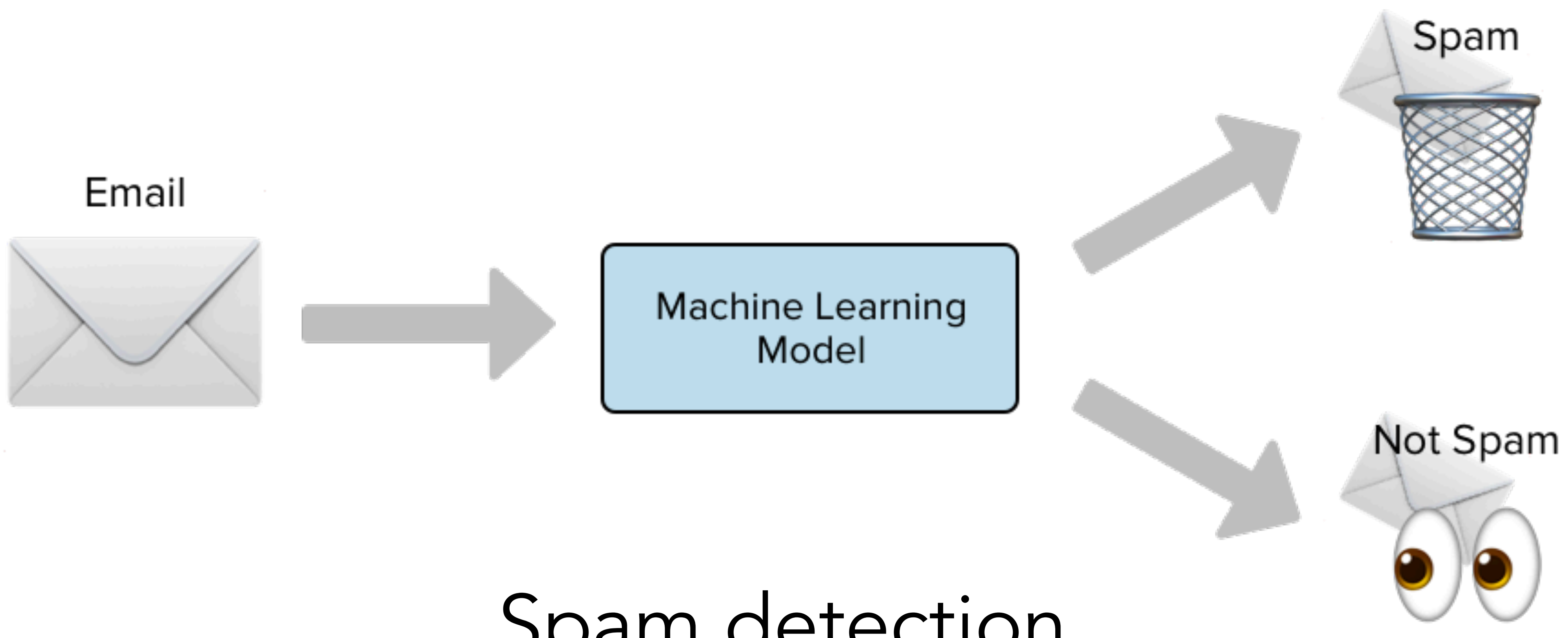
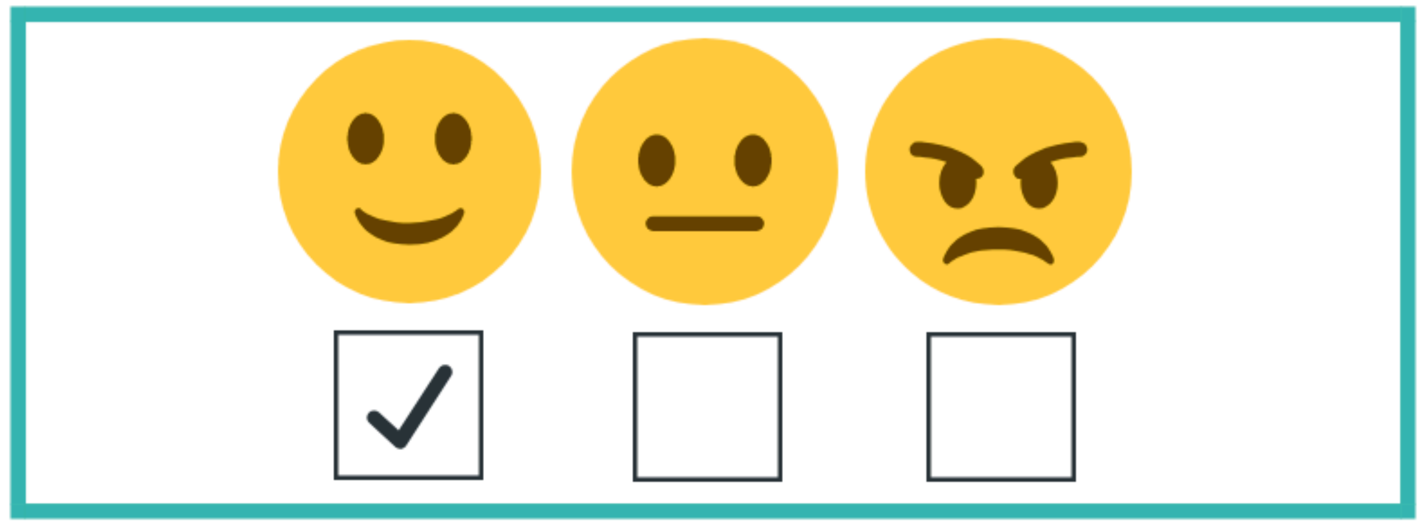
Why classify?



Spam detection



Sentiment analysis

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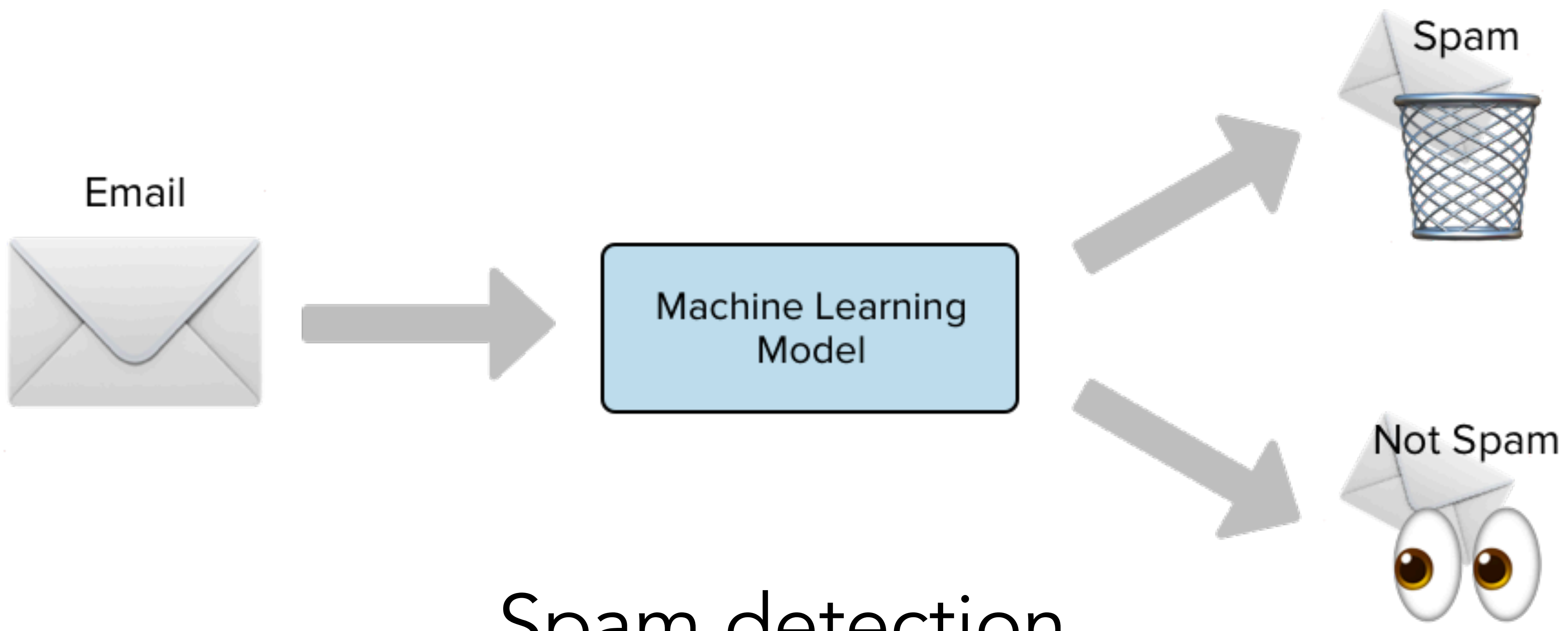

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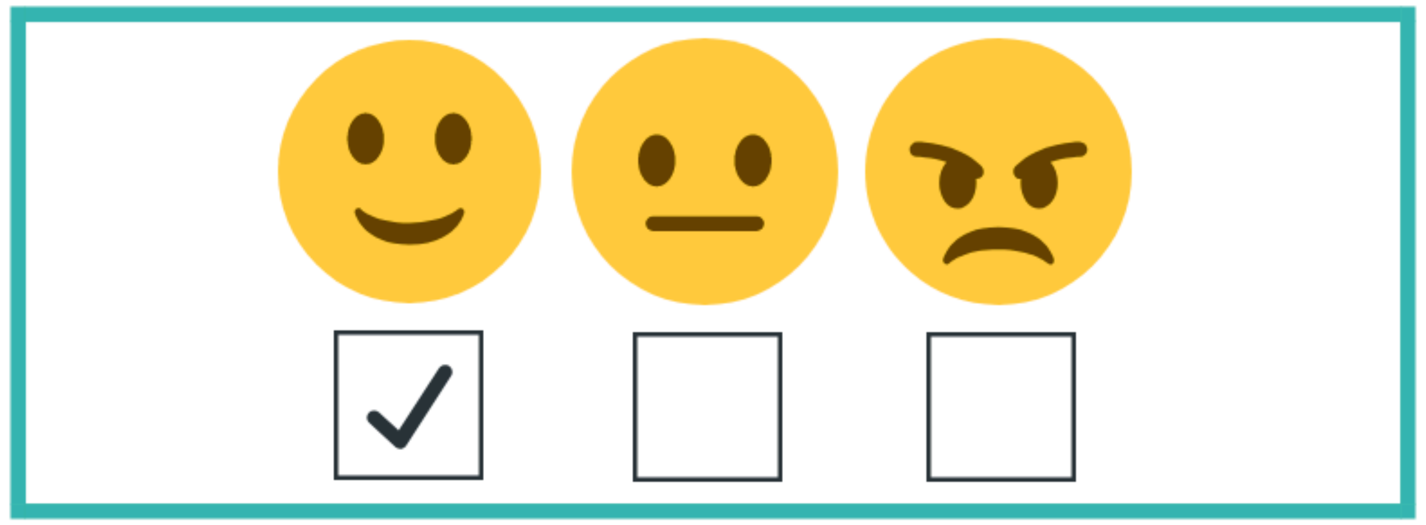
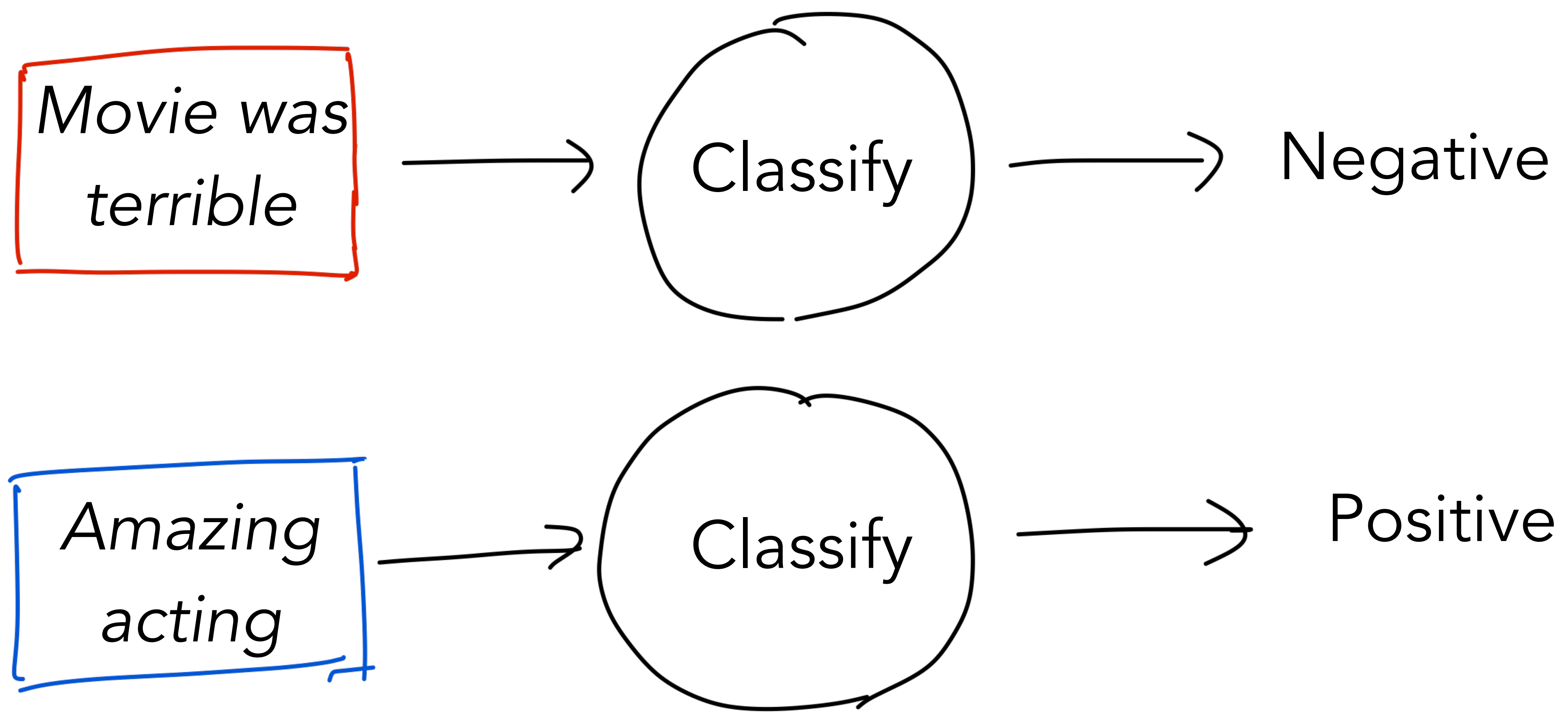


Sentiment analysis

- Authorship attribution
- Language detection
- News categorization
- …

Classification: The Task



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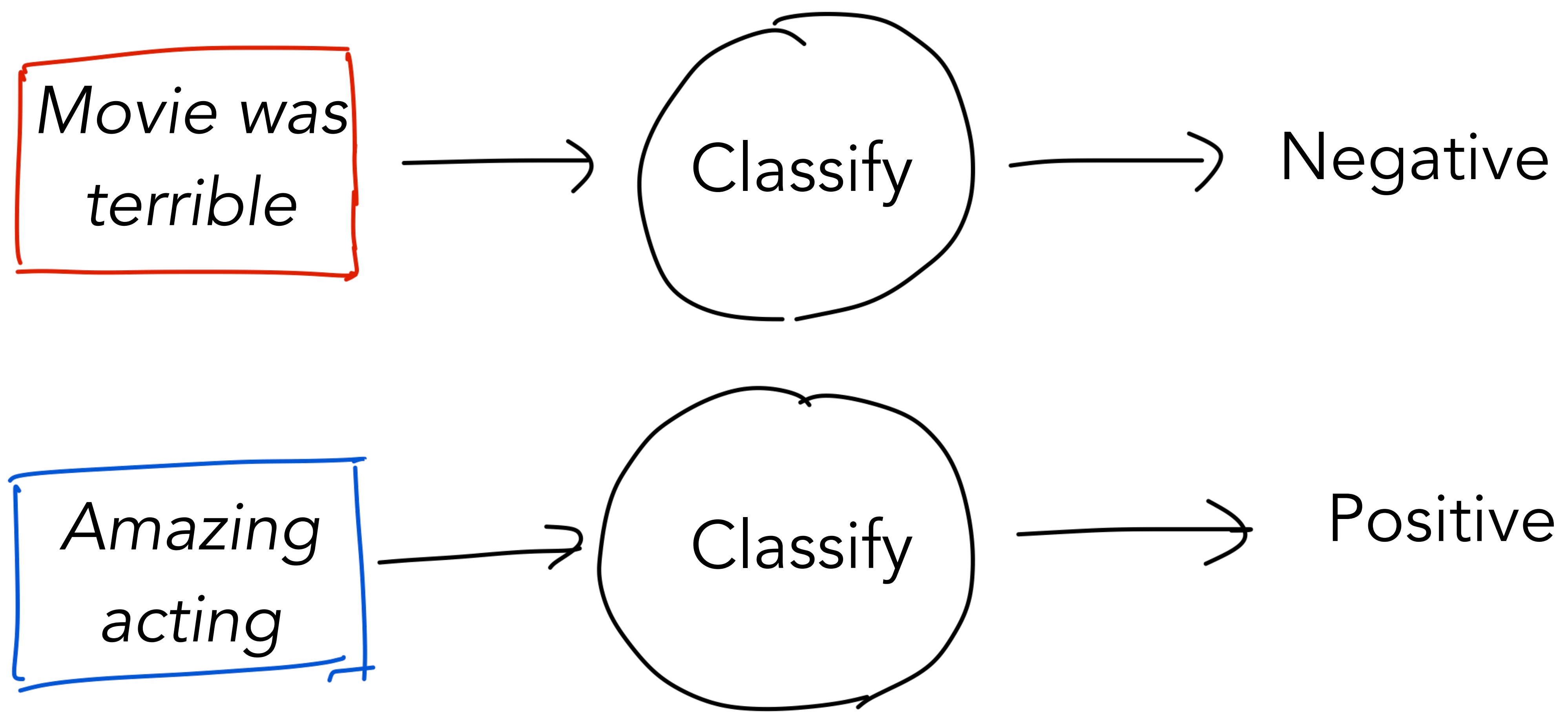

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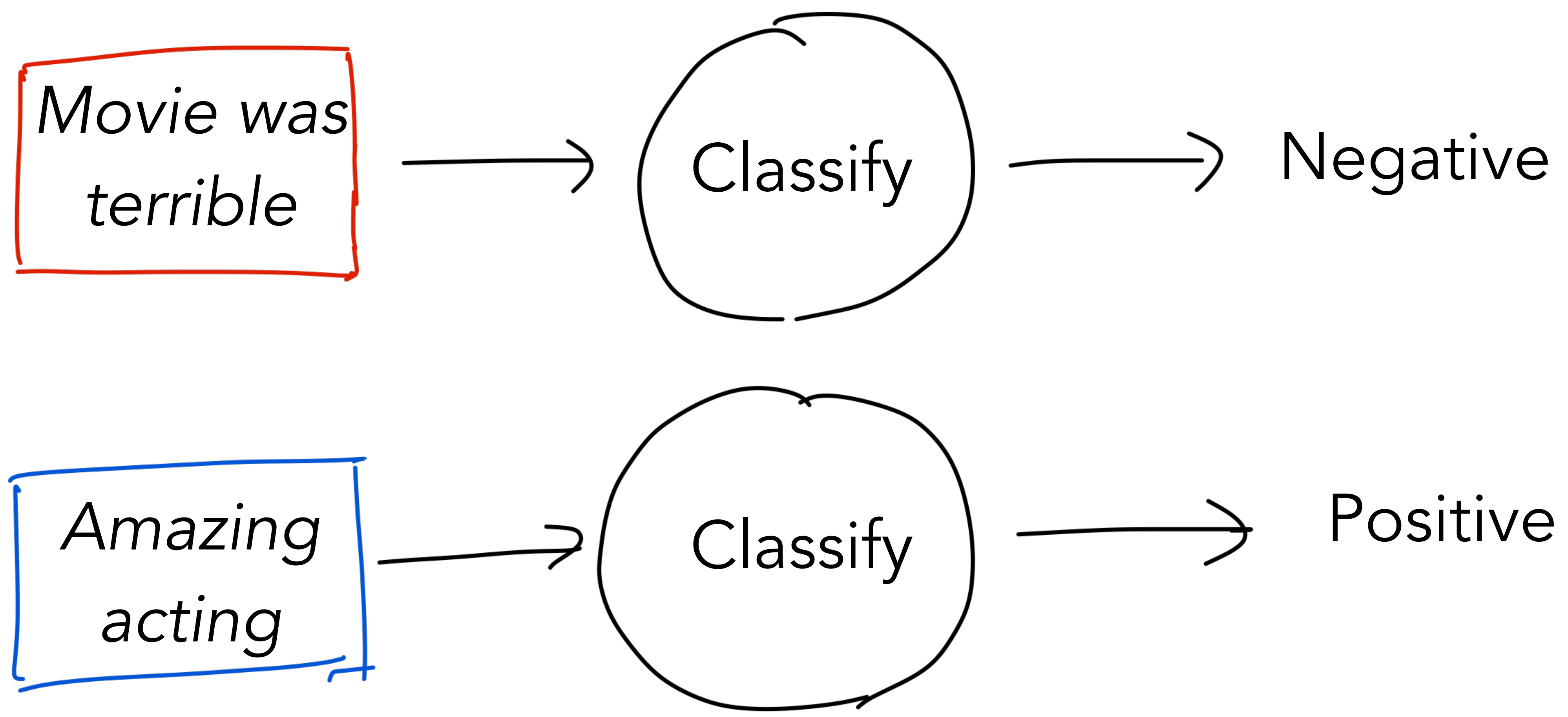

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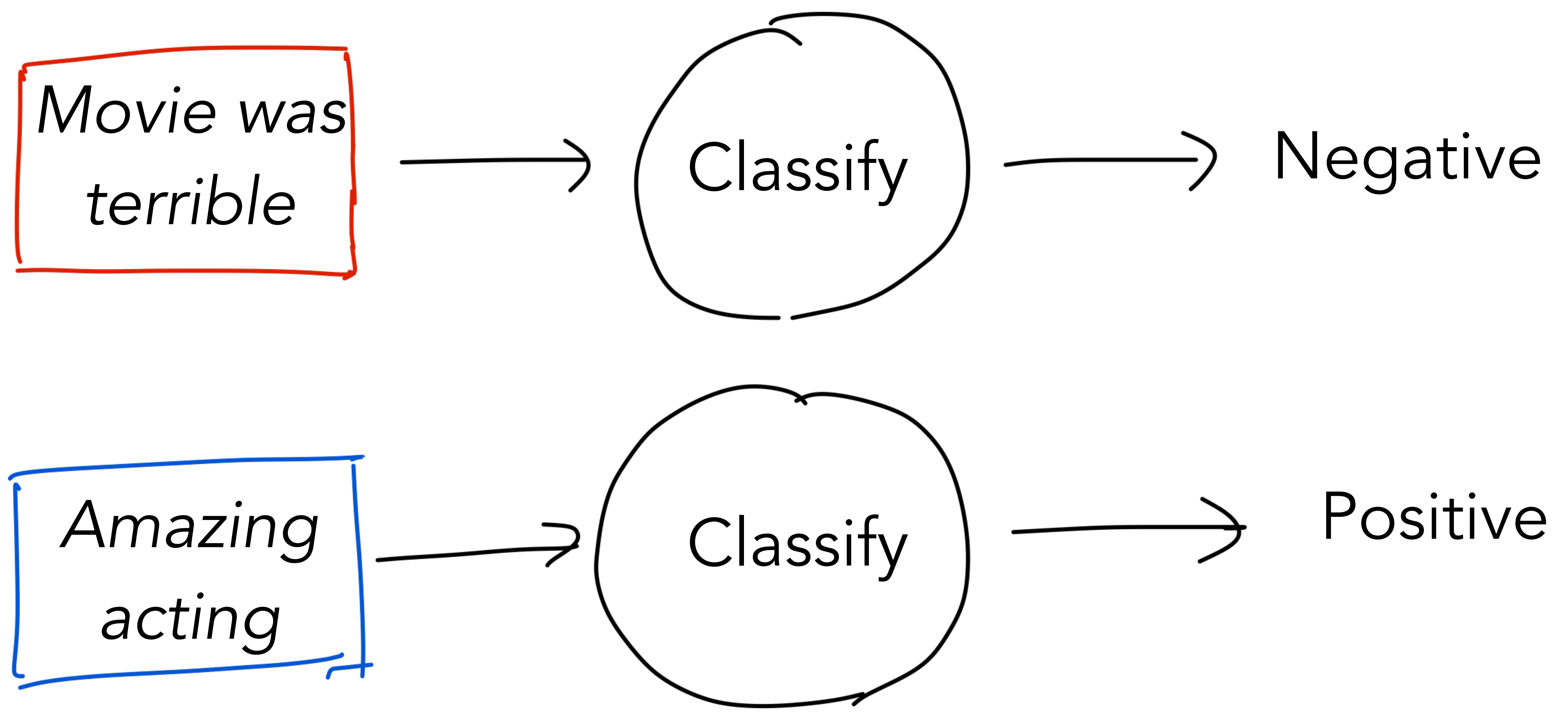

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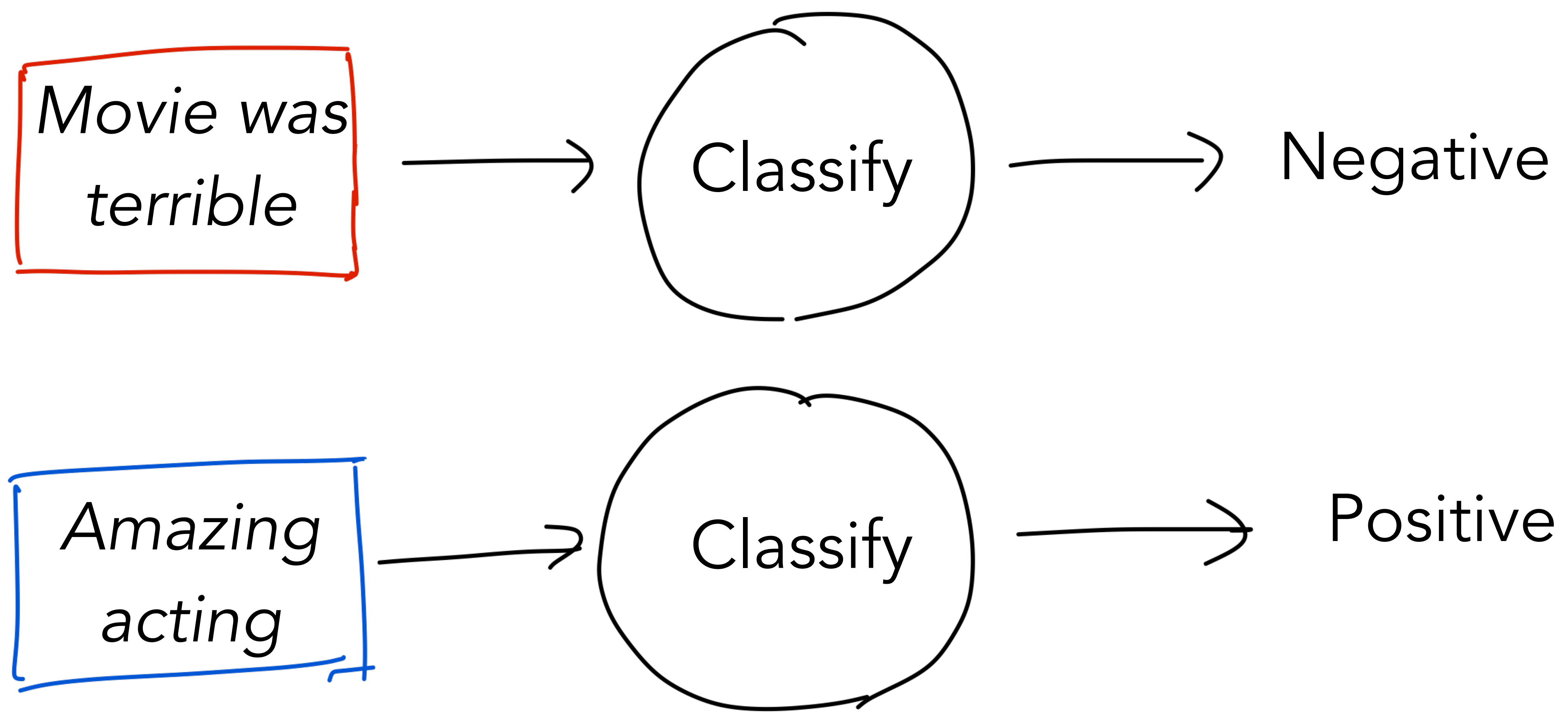

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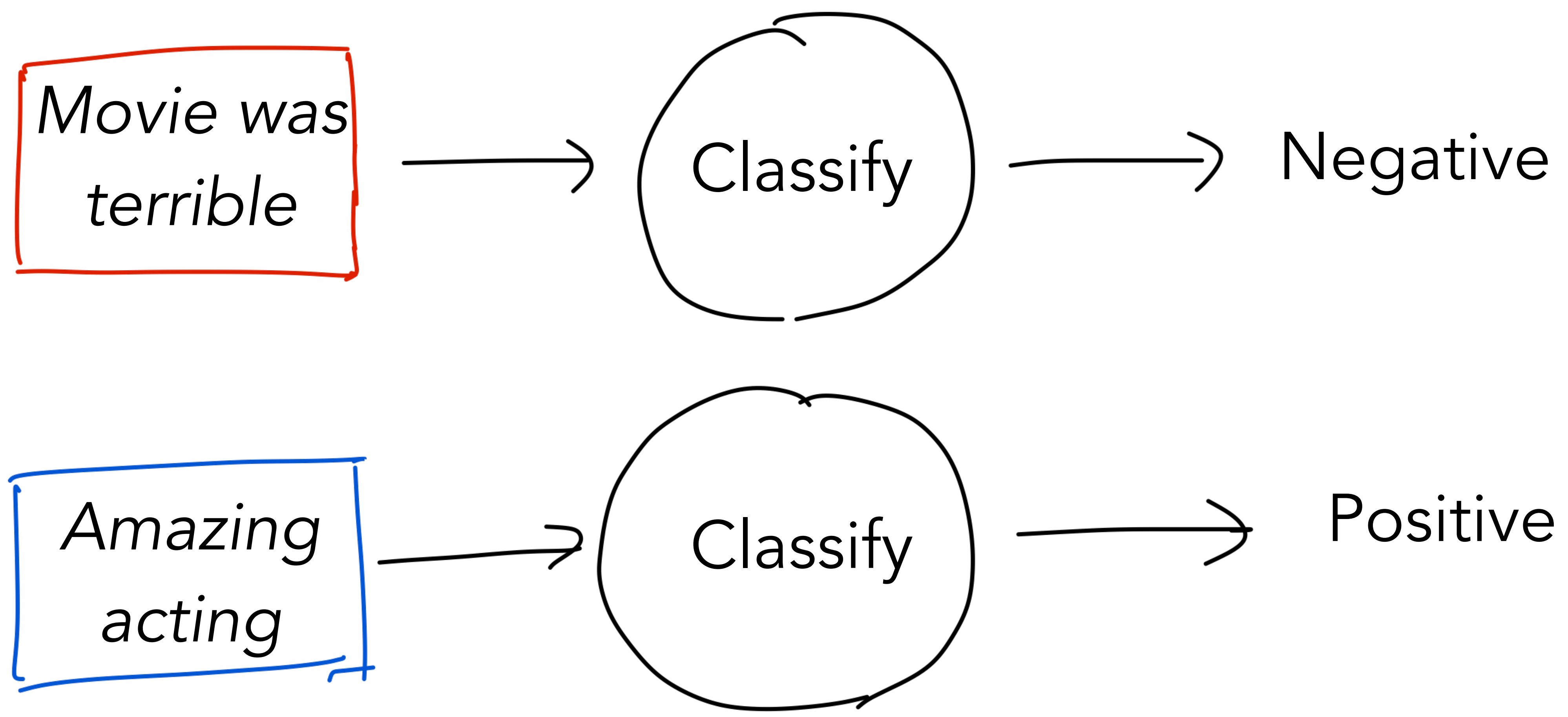

- Inputs:
  - A document d
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- Output:
  - Predicted class c for document d

Rule-based classification

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- Combinations of features on words in document, meta-data

IF there exists word w in document d such that w in [good, great, extra-ordinary, …],

THEN output Positive

IF email address ends in [*ithelpdesk.com*, *makemoney.com*, *spinthewheel.com*, …]

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VADER-Sentiment-Analysis

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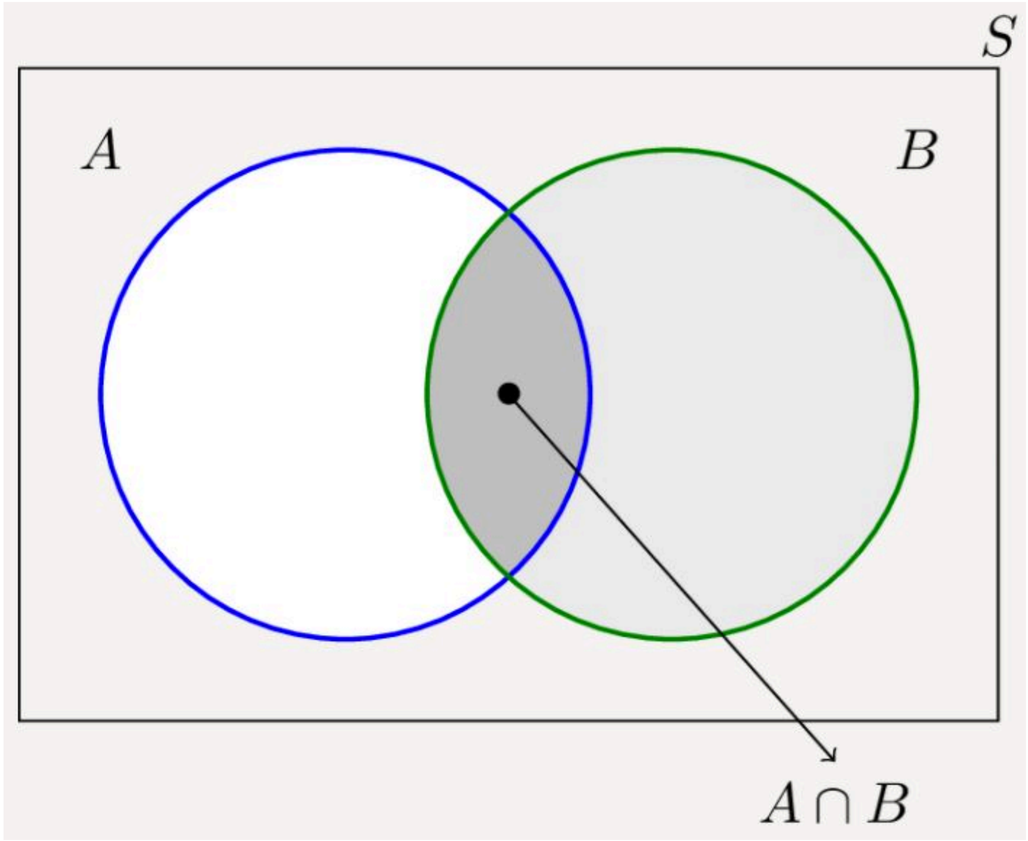
Key questions:

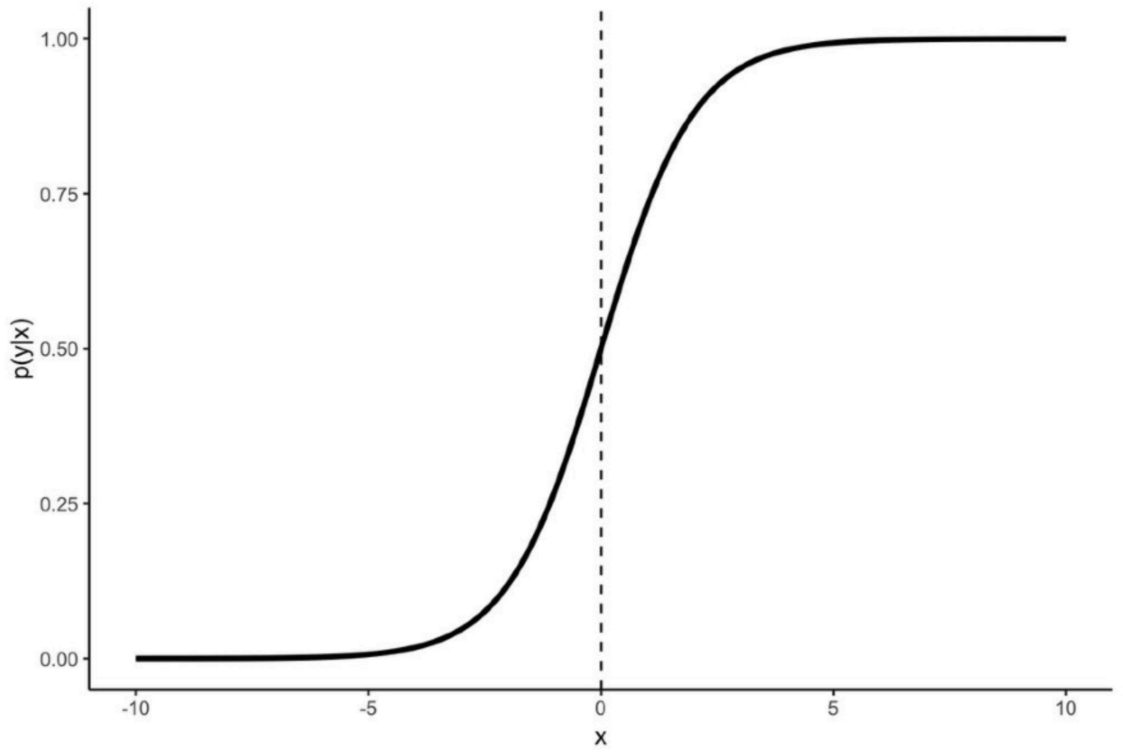
a) What is the form of F?

b) How do we learn F?

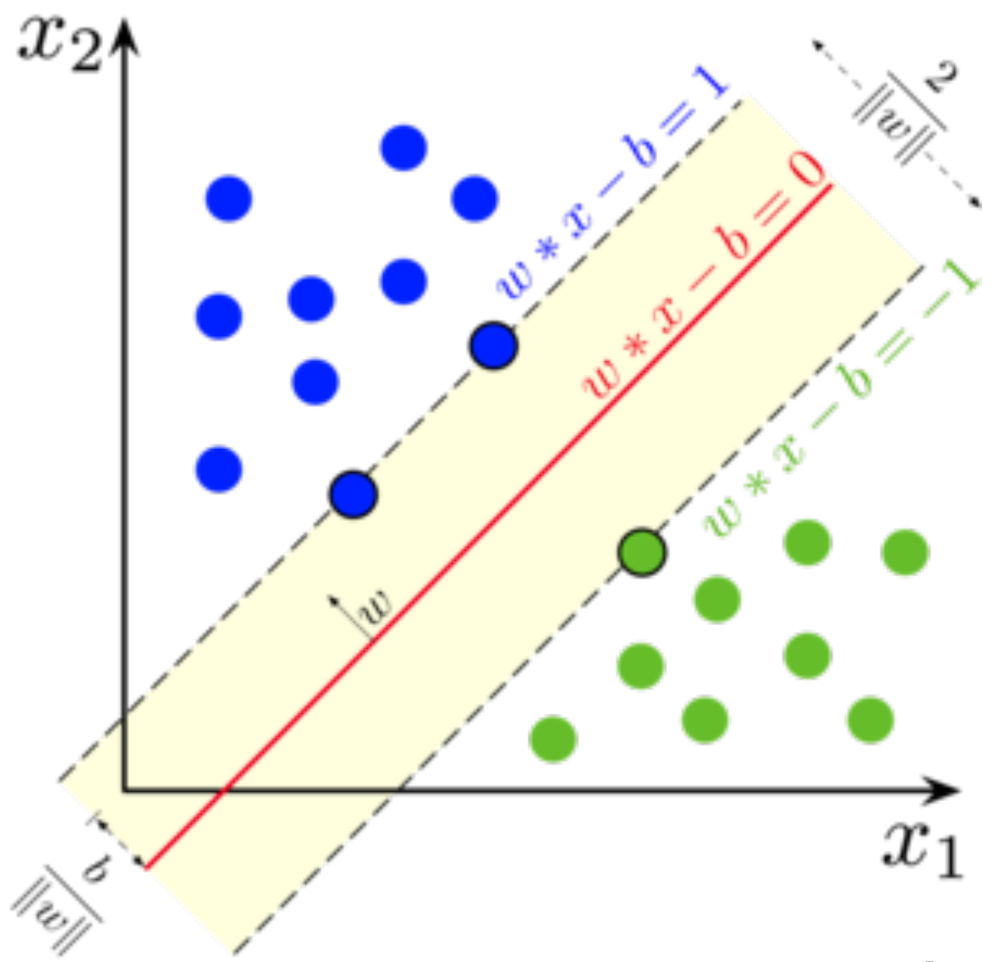
Types of supervised classifiers



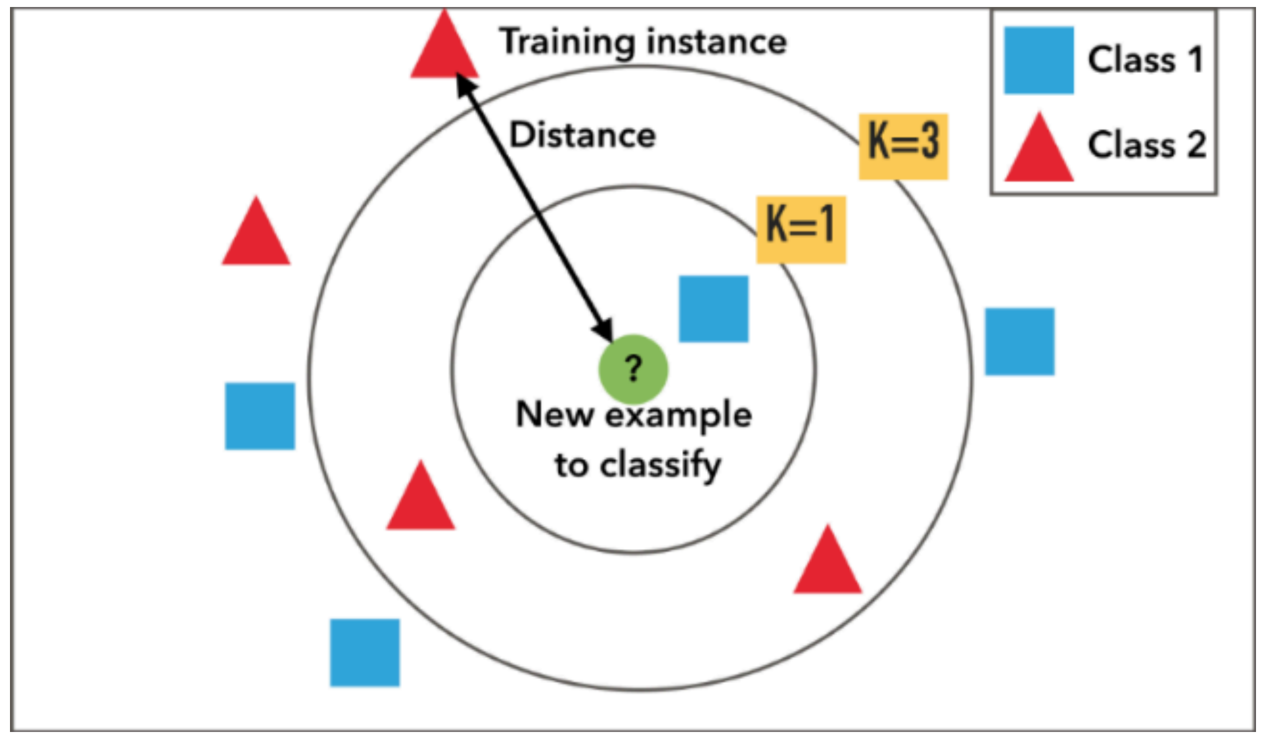
Naive Bayes



Logistic regression



Support vector machines



k-nearest neighbors

Multinomial Naive Bayes

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- Makes strong ('naive') independence assumptions

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$P(c) \longrightarrow$ Prior probability of class $\underline{c}$

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$P(c)$ → Prior probability of class c

$P(d|c)$ → Conditional probability of generating document d from class c.

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- d – document
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Maximum a posteriori (MAP) estimate → c_{MAP}

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How to represent $P(d \mid c)$?

How to represent P(d | c)?

- Option 1: represent the entire sequence of words

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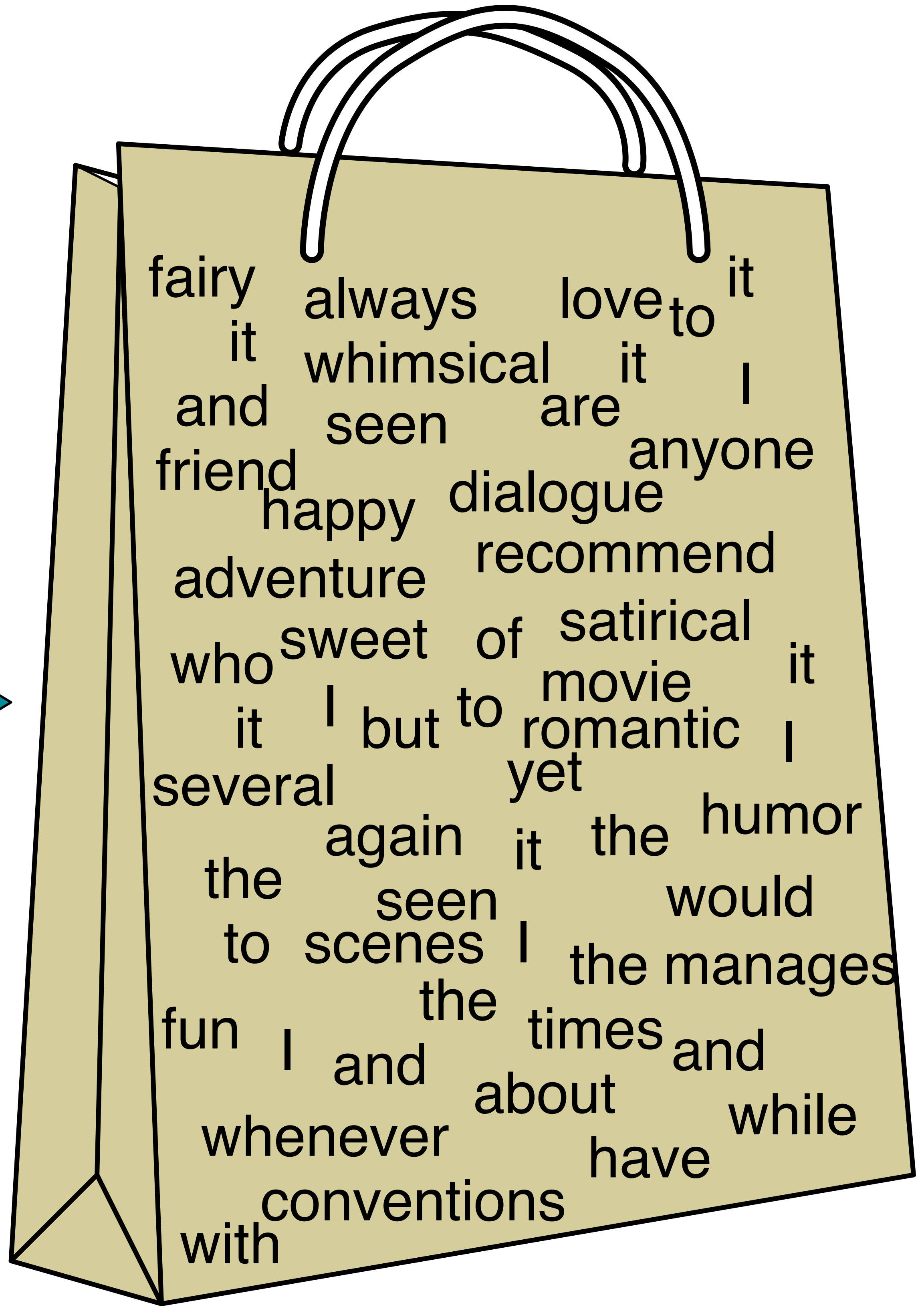
  - Assume position of each word is irrelevant (both absolute and relative)
  - $P(w_1, w_2, \ldots, w_K | c) = P(w_1 | c)P(w_2 | c)\ldots P(w_k | c)$
  - Probability of each word is *conditionally independent* of the other words given class **c**

Bag of words

I love this movie! It's sweet, but with satirical humor. The dialogue is great and the adventure scenes are fun... It manages to be whimsical and romantic while laughing at the conventions of the fairy tale genre. I would recommend it to just about anyone. I've seen it several times, and I'm always happy to see it again whenever I have a friend who hasn't seen it yet!



it	6
I	5
the	4
to	3
and	3
seen	2
yet	1
would	1
whimsical	1
times	1
sweet	1
satirical	1
adventure	1
genre	1
fairy	1
humor	1
have	1
great	1
…	…

Predicting with Naive Bayes

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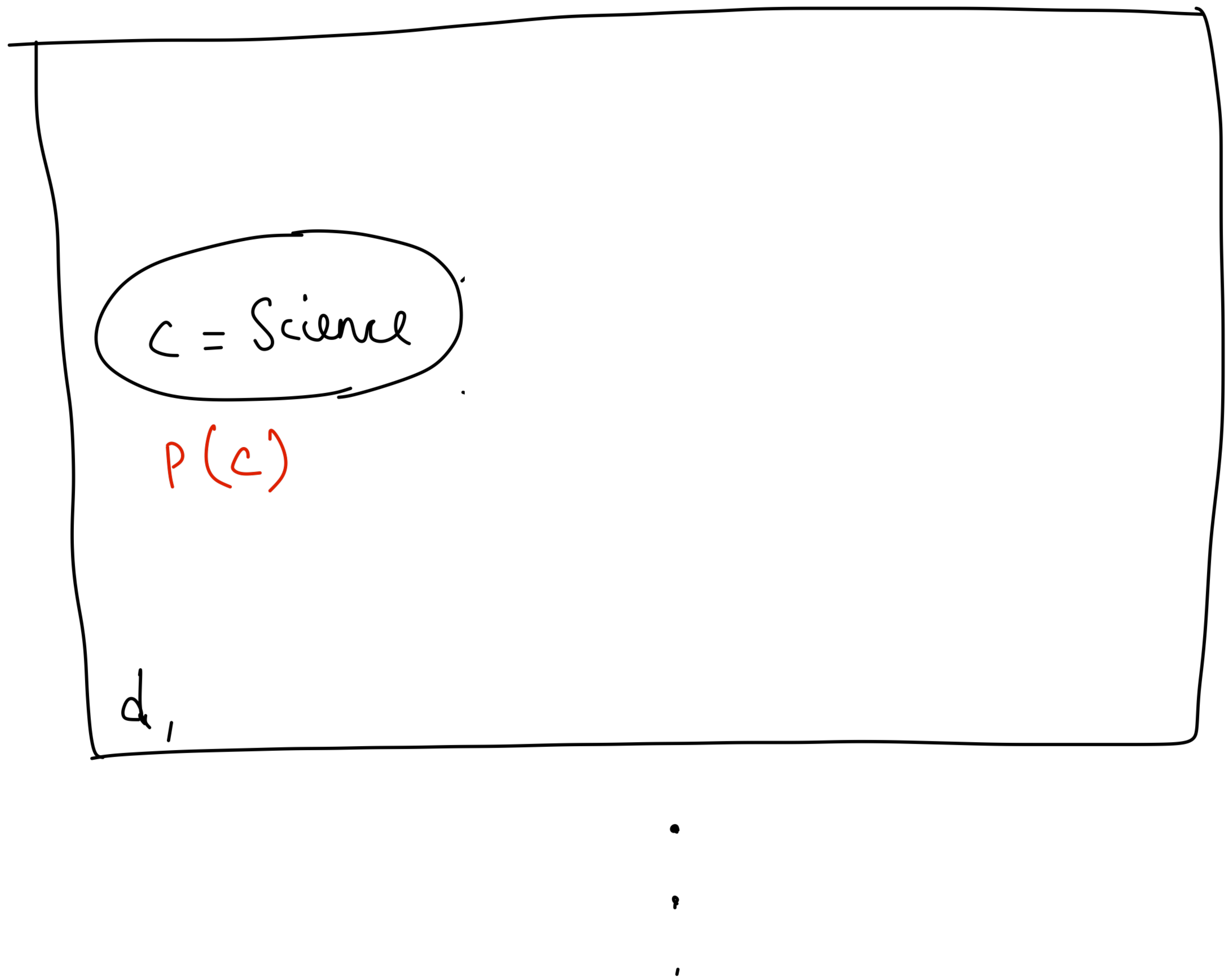
(using BOW assumption)

Naive Bayes as a generative model



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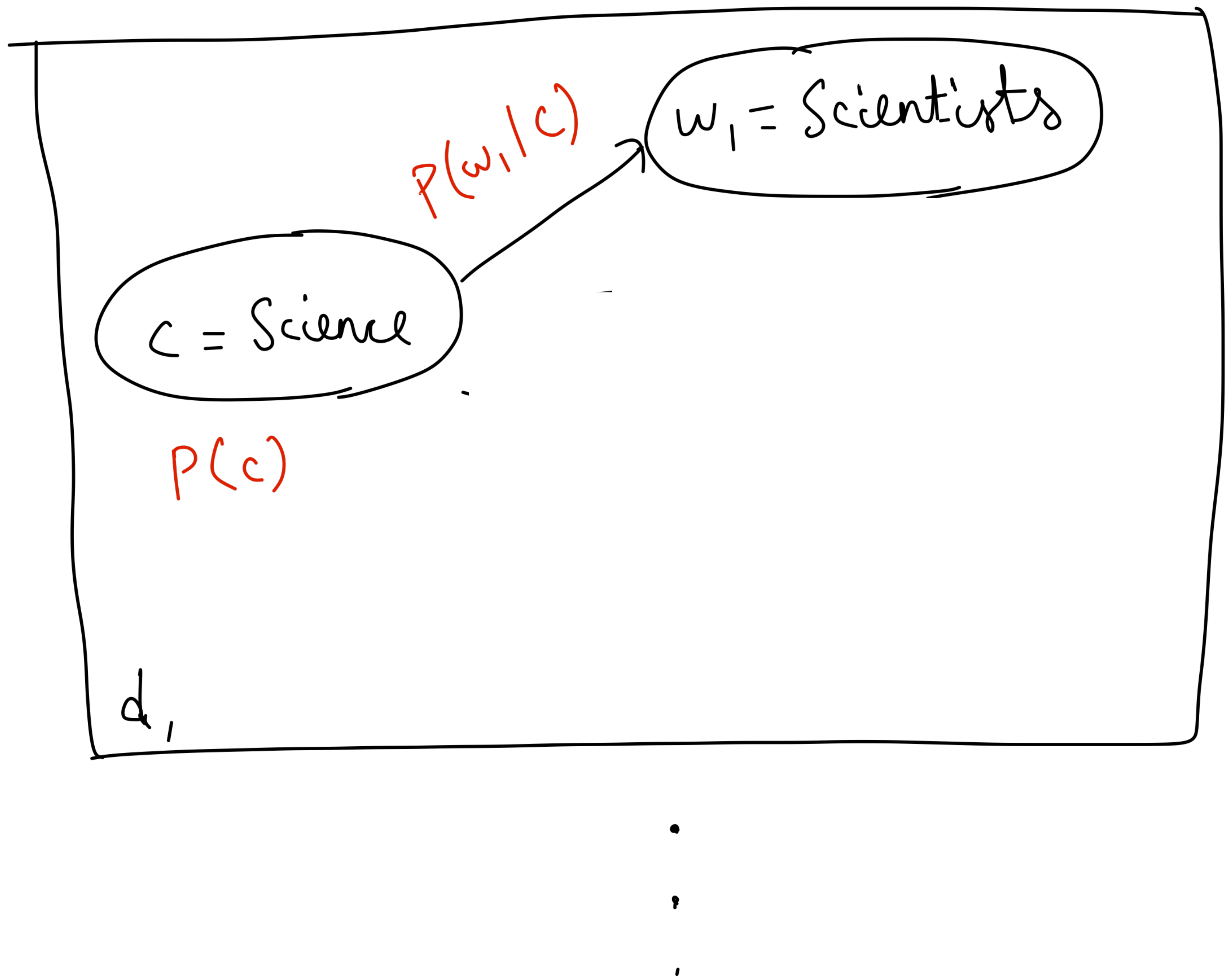

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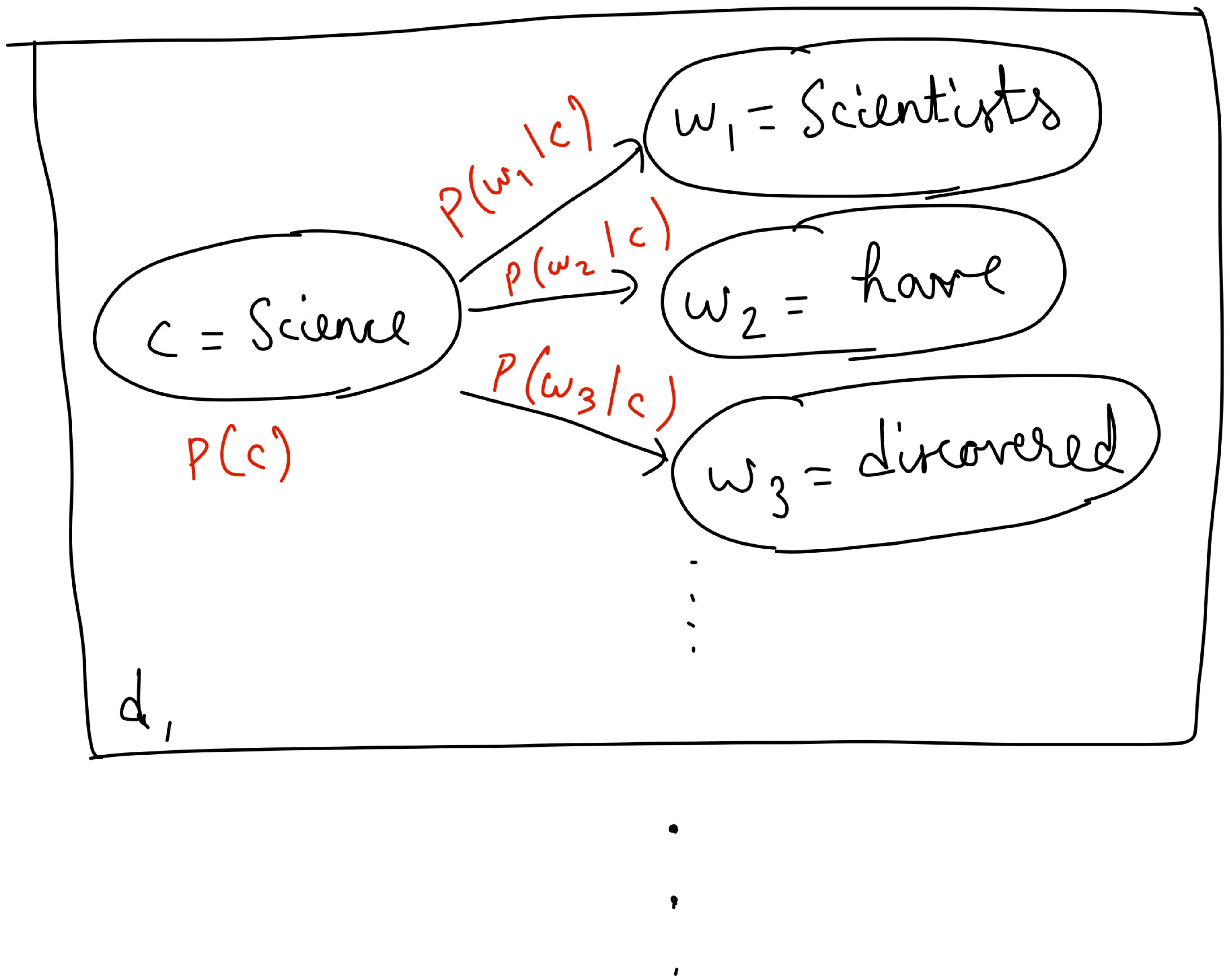

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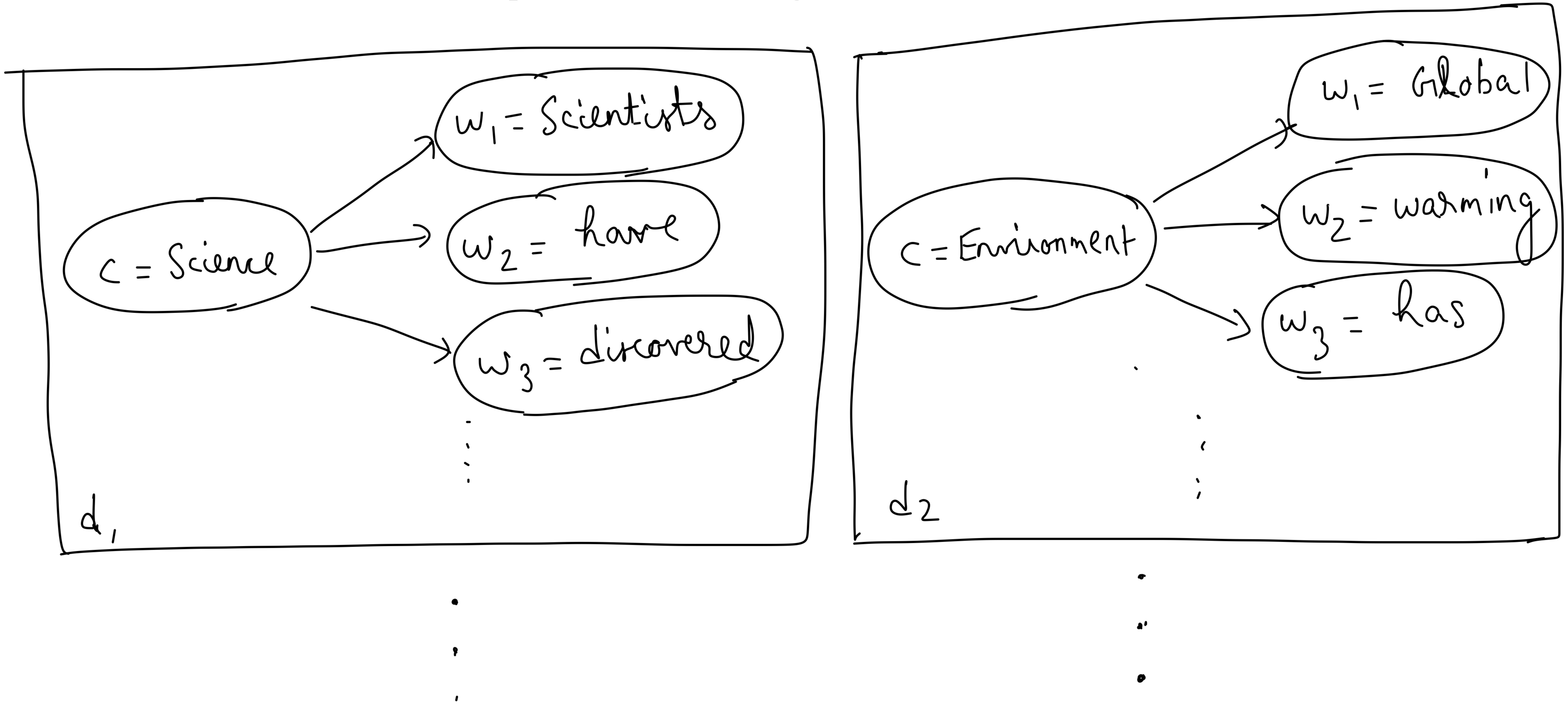

Generate the entire data set one document at a time

Estimating probabilities

$$\underset{c}{\operatorname{argmax}}\ P(c) \prod_{i=1}^{K} P(w_i \mid c)$$

Maximum likelihood estimates:

$$\hat{P}(c_j) = \frac{\text{Count}(\text{class} = c_j)}{\sum_{c} \text{Count}(\text{class} = c)}$$

← Total # of documents

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This sounds familiar…

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$$\hat{P}(w_i \mid c) = \frac{\text{Count}(w_i, c) + \alpha}{\left[\sum_{w} \text{Count}(w, c)\right] + \alpha |V|}$$

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- Simple, easy to use

- Effective in practice

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prior

Naive Bayes Example

$$\hat{P}(c) = \frac{N_c}{N}$$

$$\hat{P}(w \mid c) = \frac{count(w,c)+1}{count(c)+|V|}$$

	Doc	Words	Class
Training	1	Chinese Beijing Chinese	c
	2	Chinese Chinese Shanghai	c
	3	Chinese Macao	c
	4	Tokyo Japan Chinese	j
Test	5	Chinese Chinese Chinese Tokyo Japan	?

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Conditional Probabilities:

P(Chinese|*c*) = (5+1) / (8+6) = 6/14 = 3/7

P(Tokyo|*c*) = (0+1) / (8+6) = 1/14

P(Japan|*c*) = (0+1) / (8+6) = 1/14

P(Chinese|*j*) = (1+1) / (3+6) = 2/9

P(Tokyo|*j*) = (1+1) / (3+6) = 2/9

P(Japan|*j*) = (1+1) / (3+6) = 2/9

Naive Bayes Example

$$\hat{P}(c) = \frac{N_c}{N}$$

$$\hat{P}(w \mid c) = \frac{count(w,c)+1}{count(c)+|V|}$$

	Doc	Words	Class
Training	1	Chinese Beijing Chinese	c
	2	Chinese Chinese Shanghai	c
	3	Chinese Macao	c
	4	Tokyo Japan Chinese	j
Test	5	Chinese Chinese Chinese Tokyo Japan	?

Priors:

$P(c) = \frac{3}{4}$

$P(j) = \frac{1}{4}$

Conditional Probabilities:

P(Chinese|*c*) = (5+1) / (8+6) = 6/14 = 3/7

P(Tokyo|*c*) = (0+1) / (8+6) = 1/14

P(Japan|*c*) = (0+1) / (8+6) = 1/14

P(Chinese|*j*) = (1+1) / (3+6) = 2/9

P(Tokyo|*j*) = (1+1) / (3+6) = 2/9

P(Japan|*j*) = (1+1) / (3+6) = 2/9

Choosing a class:

$P(c|d5) \propto 3/4 * (3/7)^3 * 1/14 * 1/14 \approx 0.0003$

$P(j|d5) \propto 1/4 * (2/9)^3 * 2/9 * 2/9 \approx 0.0001$

Features

Rank	Category	Feature	Rank	Category	Feature
1	Subject	Number of capitalized words	1	Subject	Min of the compression ratio for the bz2 compressor
2	Subject	Sum of all the character lengths of words	2	Subject	Min of the compression ratio for the zlib compressor
3	Subject	Number of words containing letters and numbers	3	Subject	Min of character diversity of each word
4	Subject	Max of ratio of digit characters to all characters of each word	4	Subject	Min of the compression ratio for the lzw compressor
5	Header	Hour of day when email was sent	5	Subject	Max of the character lengths of words

(a) (b)

Spam URLs Features

1	URL	The number of all URLs in an email	1	Header	Day of week when email was sent
2	URL	The number of unique URLs in an email	2	Payload	Number of characters
3	Payload	Number of words containing letters and numbers	3	Payload	Sum of all the character lengths of words
4	Payload	Min of the compression ratio for the bz2 compressor	4	Header	Minute of hour when email was sent
5	Payload	Number of words containing only letters	5	Header	Hour of day when email was sent

Top features for spam detection

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Top features for spam detection

- In general, Naive Bayes can use any set of features, not just words:
  - URLs, email addresses, Capitalization, …
  - Domain knowledge crucial to performance

Naive Bayes and Language Models

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Class *pos*

0.1	I
0.1	love
0.01	this
0.05	fun
0.1	film

I	love	this	fun	film
0.1	0.1	.05	0.01	0.1

P(s | pos) = 0.0000005

Naive Bayes as a language model

- Which class assigns the higher probability to s?

Model pos

0.1	I
0.1	love
0.01	this
0.05	fun
0.1	film

Model neg

0.2	I
0.001	love
0.01	this
0.005	fun
0.1	film

I	love	this	fun	film
0.1	0.1	0.01	0.05	0.1
0.2	0.001	0.01	0.005	0.1

P(s|pos) ? P(s|neg)

Naive Bayes as a language model

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0.005	fun
0.1	film

I	love	this	fun	film
0.1	0.1	0.01	0.05	0.1
0.2	0.001	0.01	0.005	0.1

$P(s|pos) > P(s|neg)$

Evaluation

Evaluation

- Consider binary classification

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- Table of predictions

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Truth

Predicted

	Positive	Negative
Positive	100	5
Negative	45	100

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Truth

Confusion Matrix

Predicted

	Positive	Negative
Positive	100	5
Negative	45	100

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← Confusion Matrix

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- Ideally, we want:

Evaluation

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- Table of predictions

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	Positive	Negative
Positive	100	5
Negative	45	100

← Confusion Matrix

- Ideally, we want:

	Positive	Negative
Positive	145	0
Negative	0	105

Evaluation Metrics

Truth

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	Positive	Negative
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Evaluation Metrics

Truth

Predicted		Positive	Negative
	Positive	100	5
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- True positive: Predicted + and actual +

Evaluation Metrics

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- True negative: Predicted - and actual -
- False positive: Predicted + and actual -

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	Positive	Negative
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	Positive	Negative
Positive	100	5
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- True positive: Predicted + and actual +
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- False positive: Predicted + and actual -
- False negative: Predicted - and actual +

$$\text{Accuracy} = \frac{TP + TN}{Total} = \frac{200}{250} = 80\,\%$$

Evaluation Metrics

		Truth	
		Positive	**Negative**
Predicted	**Positive**	100	5
	Negative	45	100

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- True negative: Predicted - and actual -
- False positive: Predicted + and actual -
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Coarse metric

Evaluation Metrics

Truth

Predicted

	Positive	Negative
Positive	100	5
Negative	45	100

	Positive	Negative
Positive	100	25
Negative	25	100

$$\text{Accuracy} = \frac{TP + TN}{Total} = \frac{200}{250} = 80\,\%$$

Coarse metric

Both have same accuracy, but clearly the models are behaving very differently

Precision and Recall

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$$\text{Precision}(+) = \frac{TP}{TP + FP} \qquad \text{Precision}(-) = \frac{TN}{TN + FN}$$

- Recall: % of correct items selected

$$\text{Recall}(+) = \frac{TP}{TP + FN} \qquad \text{Recall}(-) = \frac{TN}{TN + FP}$$

F-Score

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- Combined measure using precision and recall

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F-Score

- Combined measure using precision and recall

- Harmonic mean of Precision and Recall

$$F_1 = \frac{2 \cdot \text{Precision} \cdot \text{Recall}}{\text{Precision} + \text{Recall}}$$

- Or more generally,

$$F_\beta = \frac{(1 + \beta^2) \cdot \text{Precision} \cdot \text{Recall}}{\beta^2 \cdot \text{Precision} + \text{Recall}}$$

Choosing Beta

Truth

Predicted

	Positive	Negative
Positive	200	100
Negative	50	100

$$F_\beta = \frac{(1+\beta^2) \cdot \text{Precision} \cdot \text{Recall}}{\beta^2 \cdot \text{Precision} + \text{Recall}}$$

Which value of Beta maximizes F_β for the positive class?

A. $\beta = 0.5$

B. $\beta = 1$

C. $\beta = 2$

Aggregating scores

- We now have Precision, Recall, F1 for each class
- Can we combine them for an overall score?
  - Macro-average: Compute for each class, then average
  - Micro-average: Collect predictions for all classes and jointly evaluate

Macro vs Micro average

Class 1

	Truth: yes	Truth: no
Classifier: yes	10	10
Classifier: no	10	970

Class 2

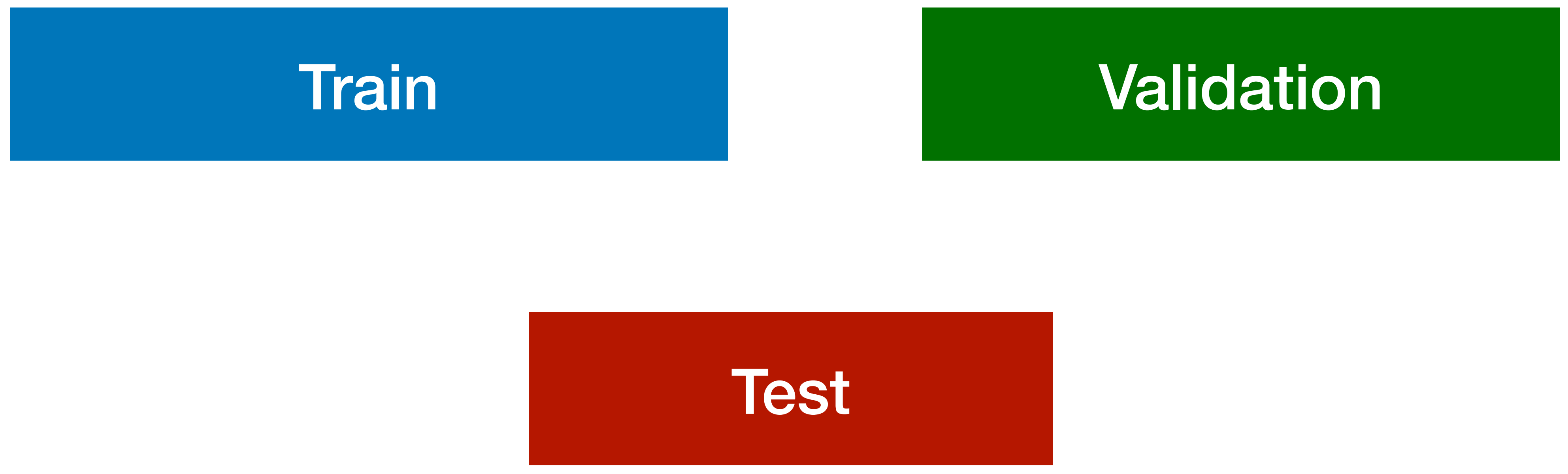
	Truth: yes	Truth: no
Classifier: yes	90	10
Classifier: no	10	890

Micro Ave. Table

	Truth: yes	Truth: no
Classifier: yes	100	20
Classifier: no	20	1860

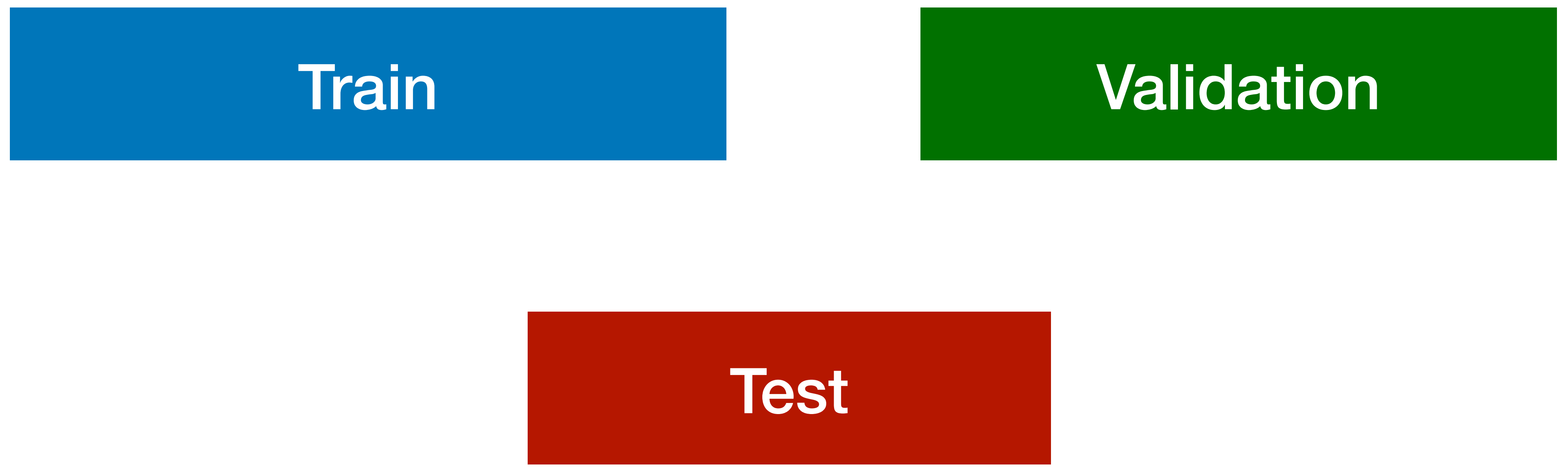
- Macroaveraged precision: $(0.5 + 0.9)/2 = 0.7$
- Microaveraged precision: $100/120 = .83$
- Microaveraged score is dominated by score on common classes

Validation



Validation

- Choose a metric: Precision/Recall/F1



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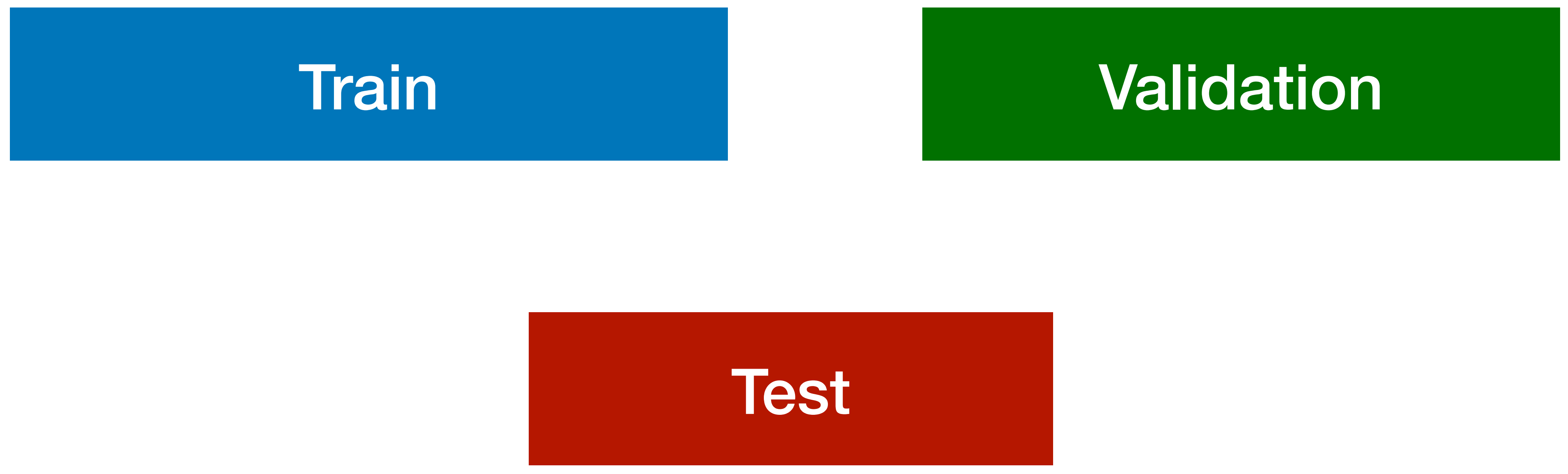

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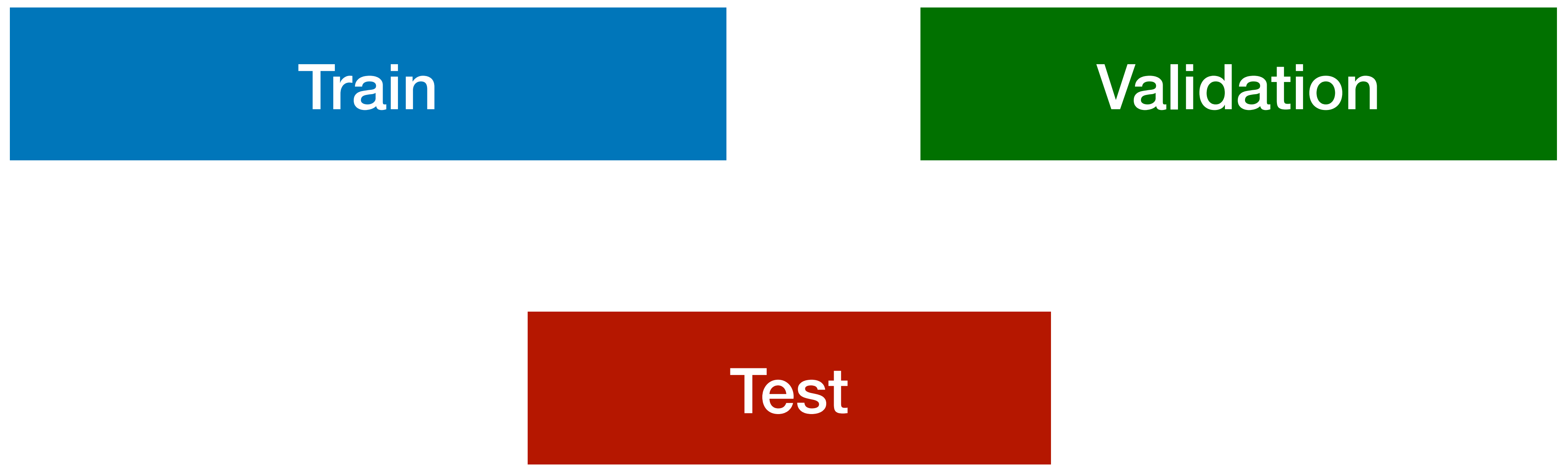

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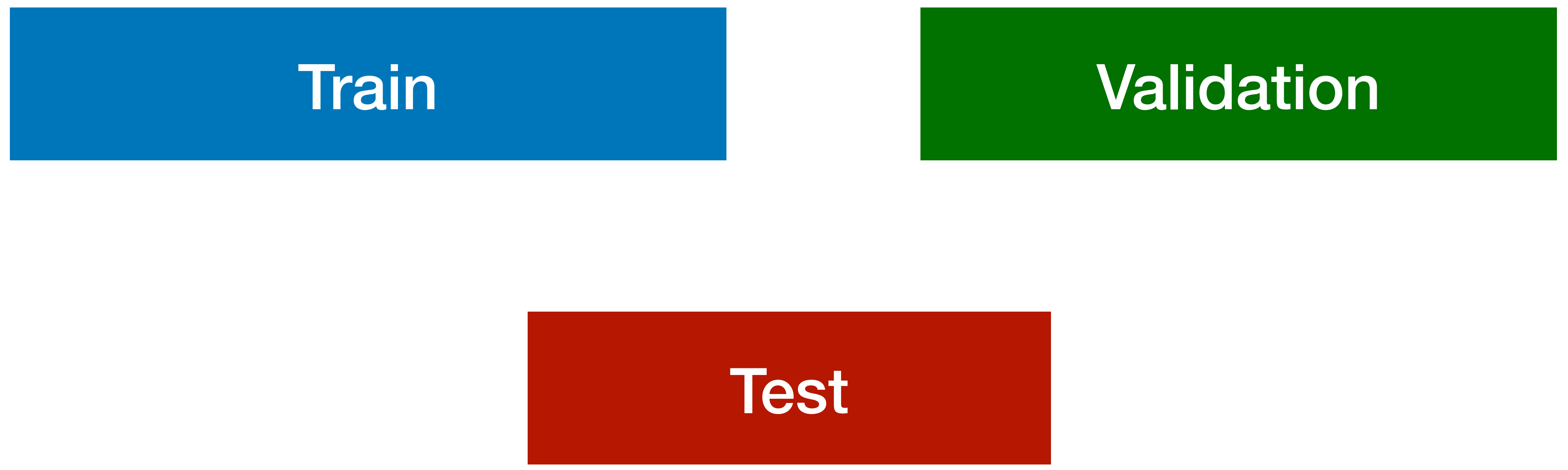

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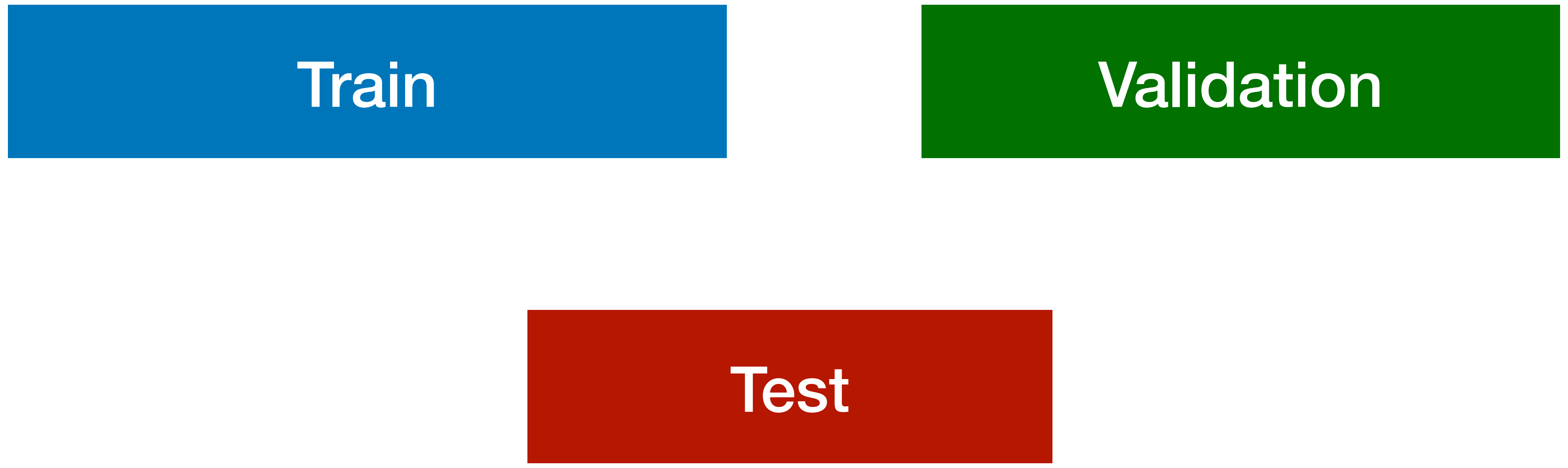

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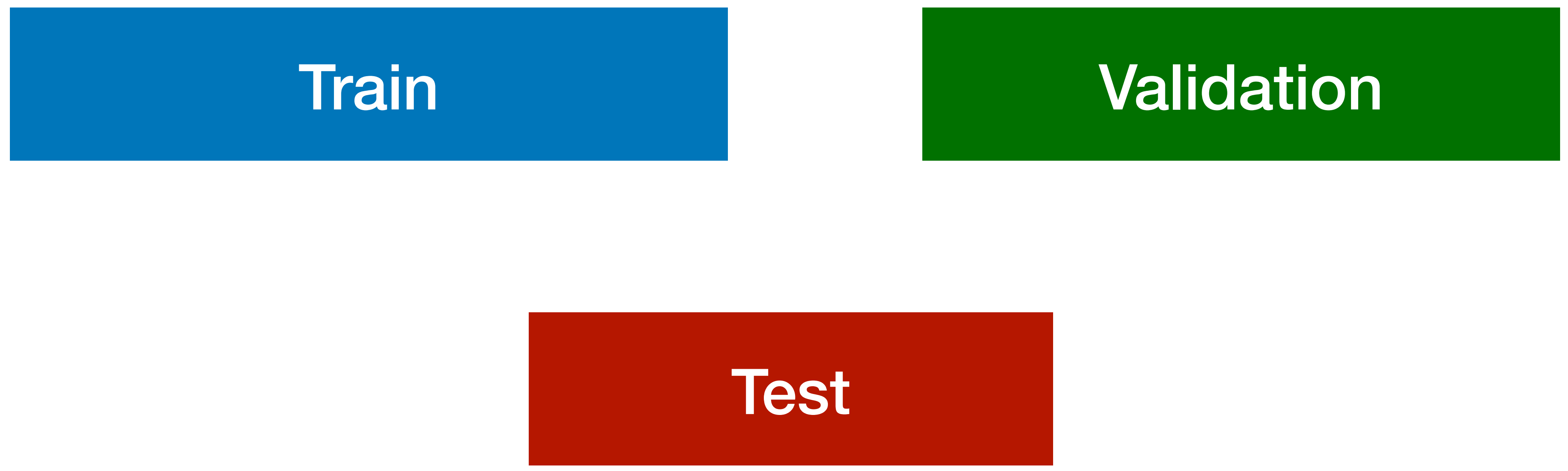

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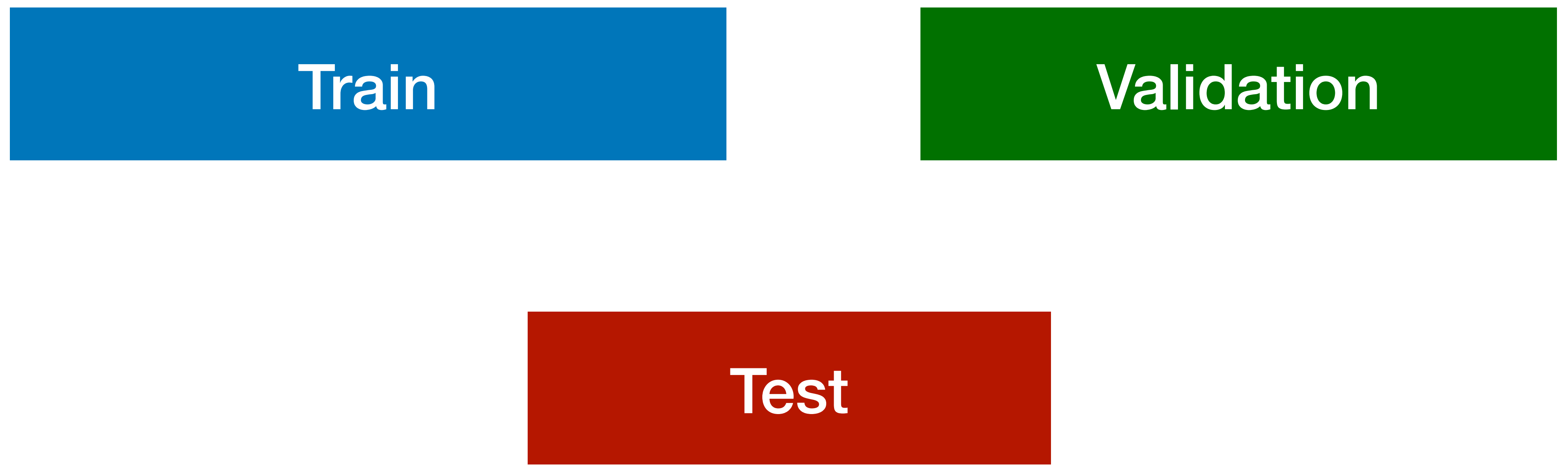

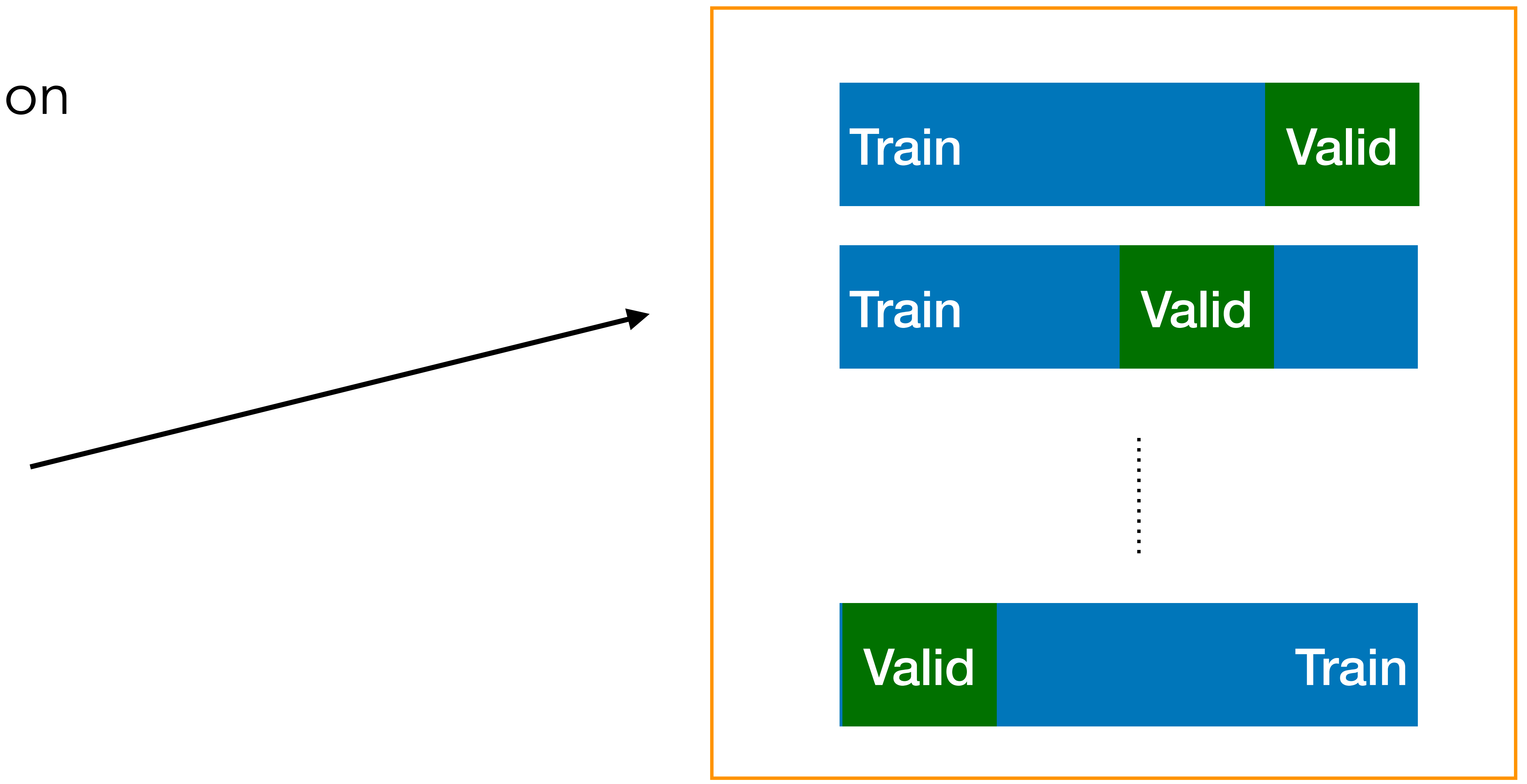

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- Cross-validation:
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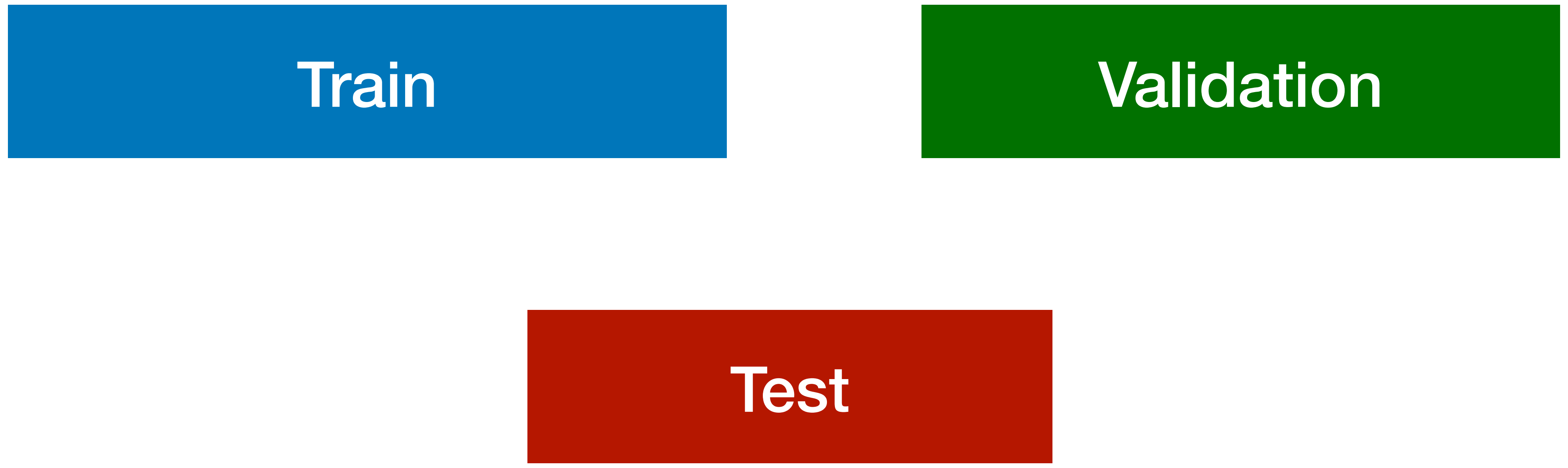
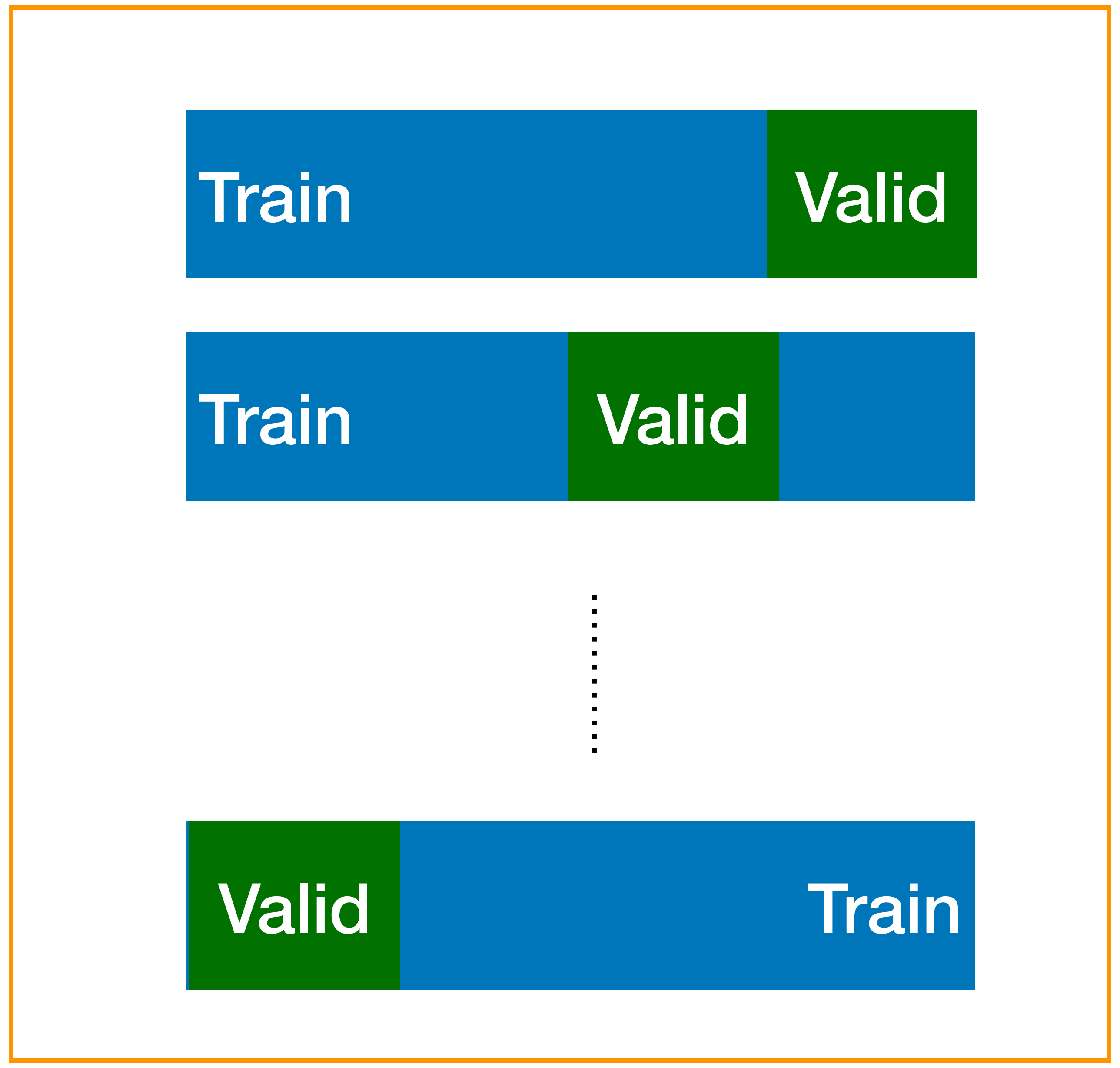
  - Reduces bias due to sampling errors





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  If assumed independence is correct, this is the 'Bayes optimal' classifier
- A good dependable baseline for text classification

  However, other classifiers can give better accuracy

Practical Naive Bayes

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  - Rule-based classifiers might work well too
- Medium size datasets:
  - More advanced classifiers might perform better (e.g. SVM, logistic regression)
- Large datasets:
  - Naive Bayes becomes competitive again (although most classifiers work well)

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- XOR problem: Naive Bayes cannot learn a decision boundary
- Both variables are jointly required to predict class

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$$\hat{P}(w_i \mid \tilde{c}_j) = \frac{\sum_{c \neq c_j} \text{Count}(w_i, c)}{\sum_{c \neq c_j} \sum_{w} \text{Count}(w, c)}$$

→ Count # times word occurs in classes other than c

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(assuming ϵ added for smoothing)

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Weight magnitude errors

- Classes with larger weights are preferred
- 10 documents with class=MA and "Boston" occurring once each
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Failings of Naive Bayes (3)

Weight magnitude errors

- Classes with larger weights are preferred
- 10 documents with class=MA and "Boston" occurring once each
- 10 documents with class=CA and "San Francisco" occurring once each
- New document: "Boston Boston Boston San Francisco San Francisco"

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- 10 documents with class=MA and “Boston” occurring once each
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- New document: “Boston Boston Boston San Francisco San Francisco”

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- 10 documents with class=MA and "Boston" occurring once each
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- New document: "Boston Boston Boston San Francisco San Francisco"

$$P(\text{class} = CA \mid \text{document}) \; ? \; P(\text{class} = MA \mid \text{document})$$

(assuming ϵ added for smoothing)

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- Model is now just max of sum of weights