COS 484/584

L4: Logistic regression

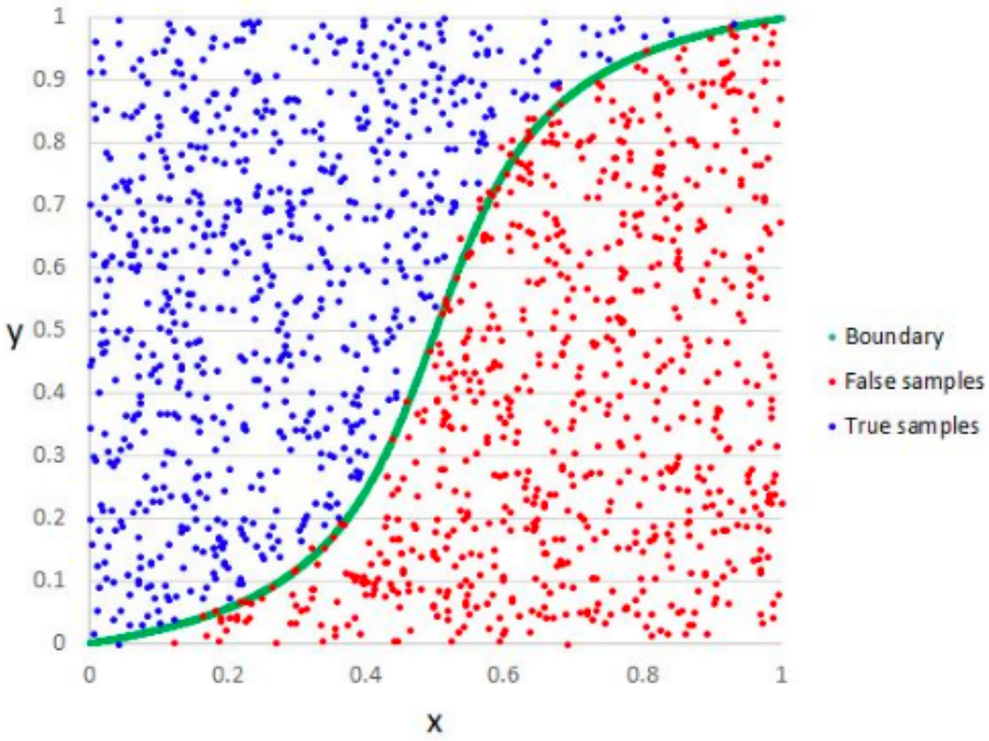
Spring 2021

Last class

- Supervised classification:
  - Document to classify, d
  - Set of classes, $C = \{c_1, c_2, \ldots, c_k\}$
- Naive Bayes:

$$\hat{c} = \underset{c}{\operatorname{argmax}}\ P(c)\ P(d|c)$$

Logistic Regression



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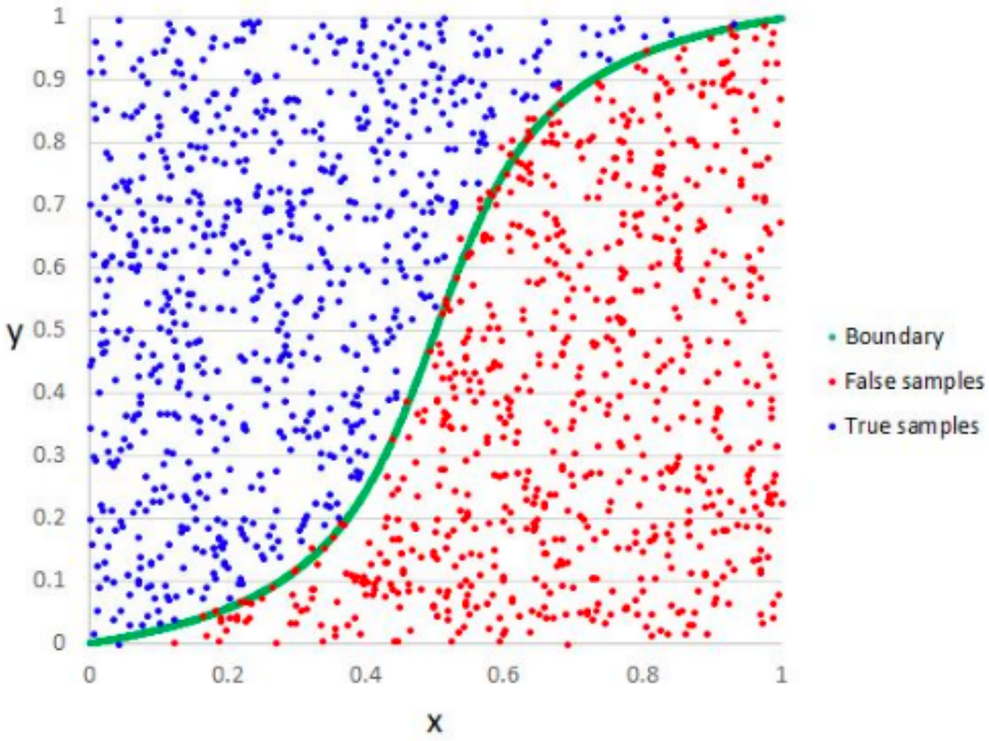

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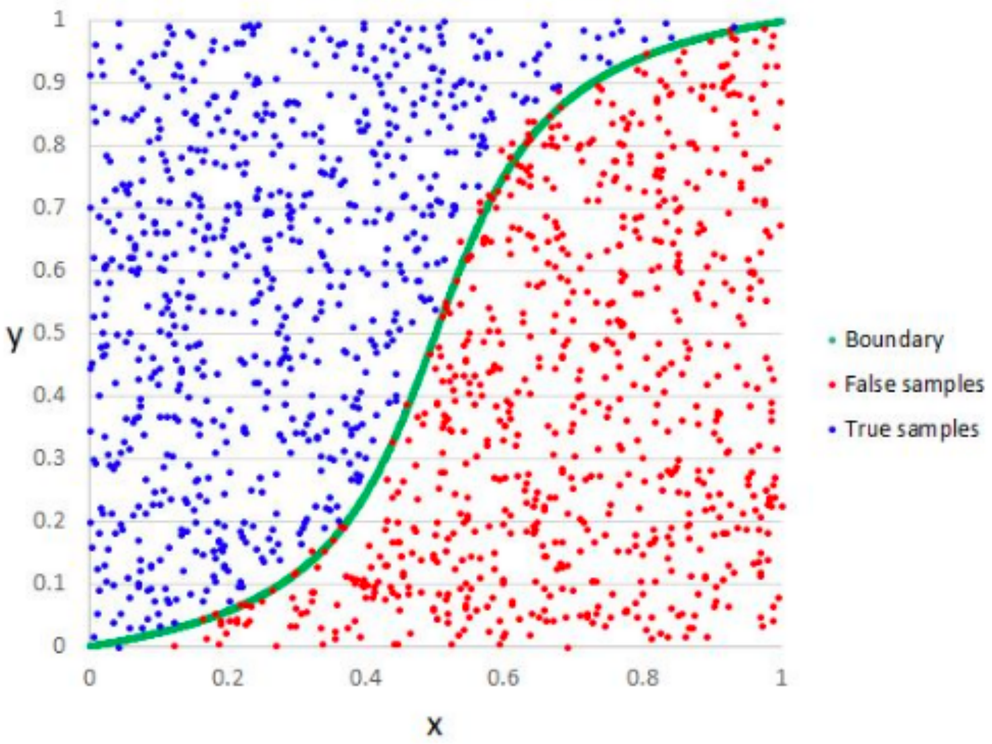
Logistic Regression



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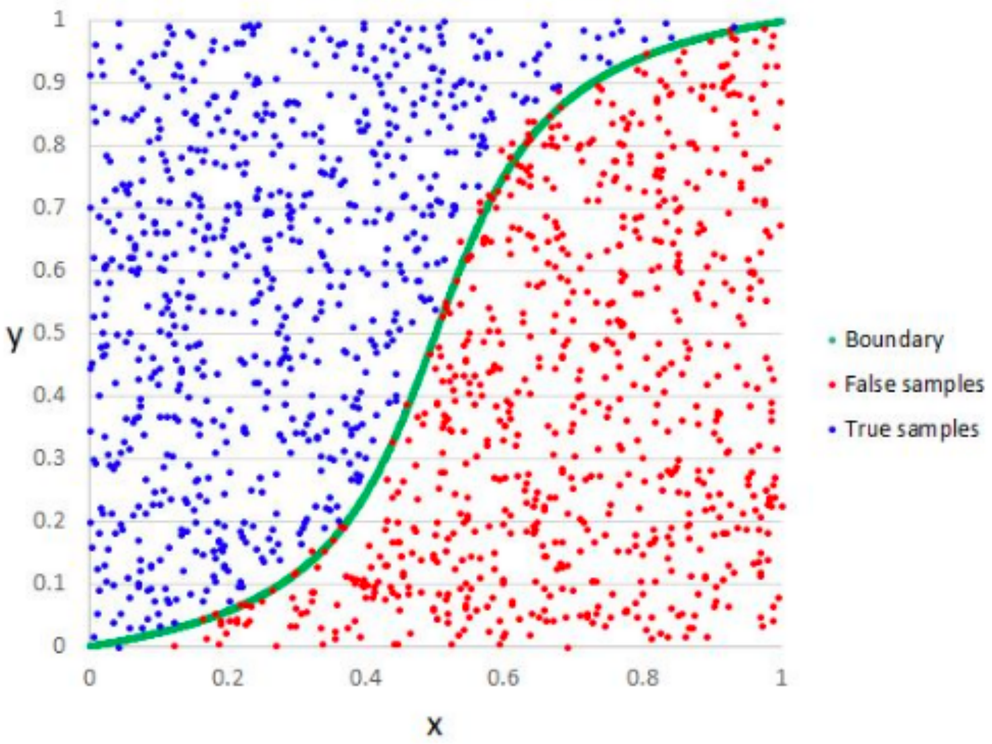

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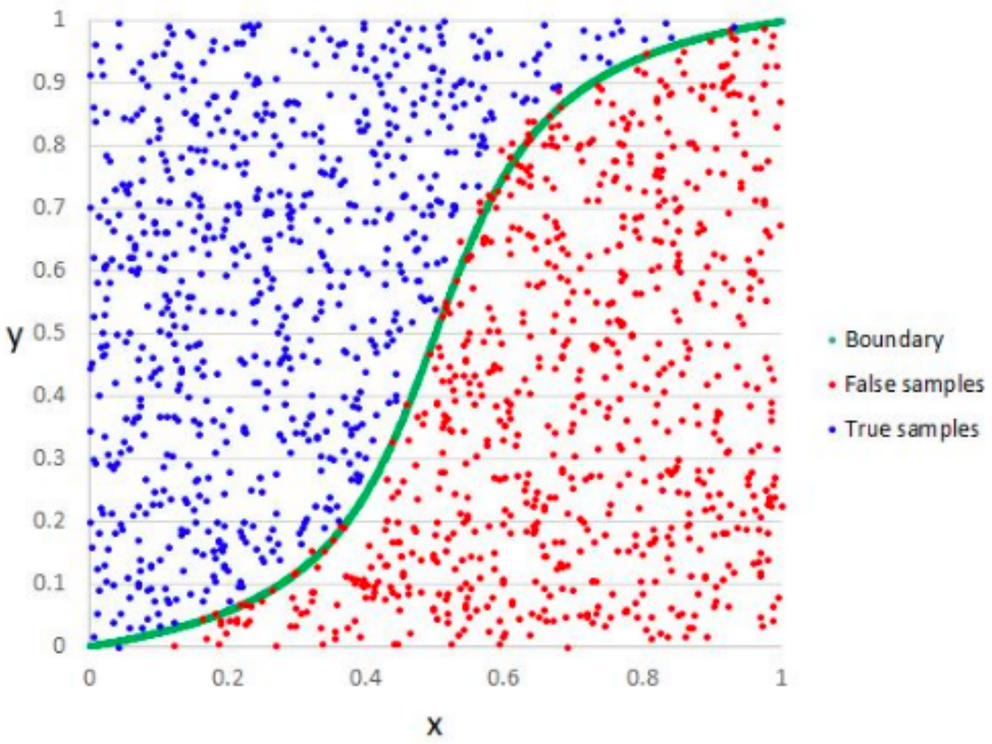

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- Binary (two classes) or multinomial (>2 classes)

Discriminative Model

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- Naive Bayes: *generative* model

Discriminative Model

- Logistic Regression: $\hat{c} = \underset{c}{\operatorname{argmax}} \; P(c|d)$
- Naive Bayes: $\hat{c} = \underset{c}{\operatorname{argmax}} \; P(c)\,P(d|c)$

Using Logistic Regression

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- Train phase: Learn the **parameters** of the model to minimize **loss function**
- Test phase: Apply **parameters** to predict class given a new input x

1. Feature representation

- Input observation: $x^{(i)}$
- Feature vector: $[x_1, x_2, \ldots, x_d]$
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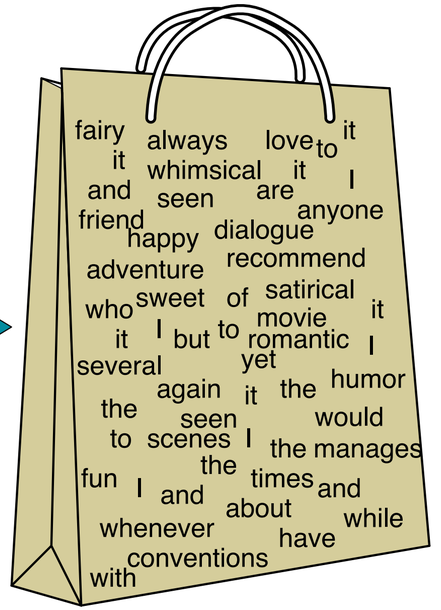
I love this movie! It's sweet, but with satirical humor. The dialogue is great and the adventure scenes are fun... It manages to be whimsical and romantic while laughing at the conventions of the fairy tale genre. I would recommend it to just about anyone. I've seen it several times, and I'm always happy to see it again whenever I have a friend who hasn't seen it yet!

$x^{(i)}$



it	6
I	5
the	4
to	3
and	3
seen	2
yet	1
would	1
whimsical	1
times	1
sweet	1
satirical	1
adventure	1
genre	1
fairy	1
humor	1
have	1
great	1

$[x_1, x_2, \ldots, x_d]$

Bag of words

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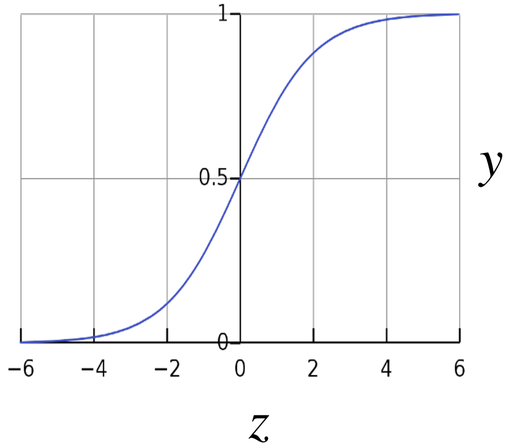

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- Given an instance, x: $z = \sum_{i=1}^{d} w_i x_i + b$ or $z = w \cdot x + b$
- Therefore, $y = \dfrac{e^{w \cdot x + b}}{1 + e^{w \cdot x + b}}$

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$$y = \frac{e^{w \cdot x + b}}{1 + e^{w \cdot x + b}} = \frac{e^{w \cdot x}}{1 + e^{w \cdot x + b}} e^{b}$$

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$$\begin{aligned} P(y = 0) &= 1 - \sigma(w \cdot x + b) \\ &= 1 - \frac{1}{1 + e^{-(w \cdot x + b)}} = \frac{e^{-(w \cdot x + b)}}{1 + e^{-(w \cdot x + b)}} \end{aligned}$$

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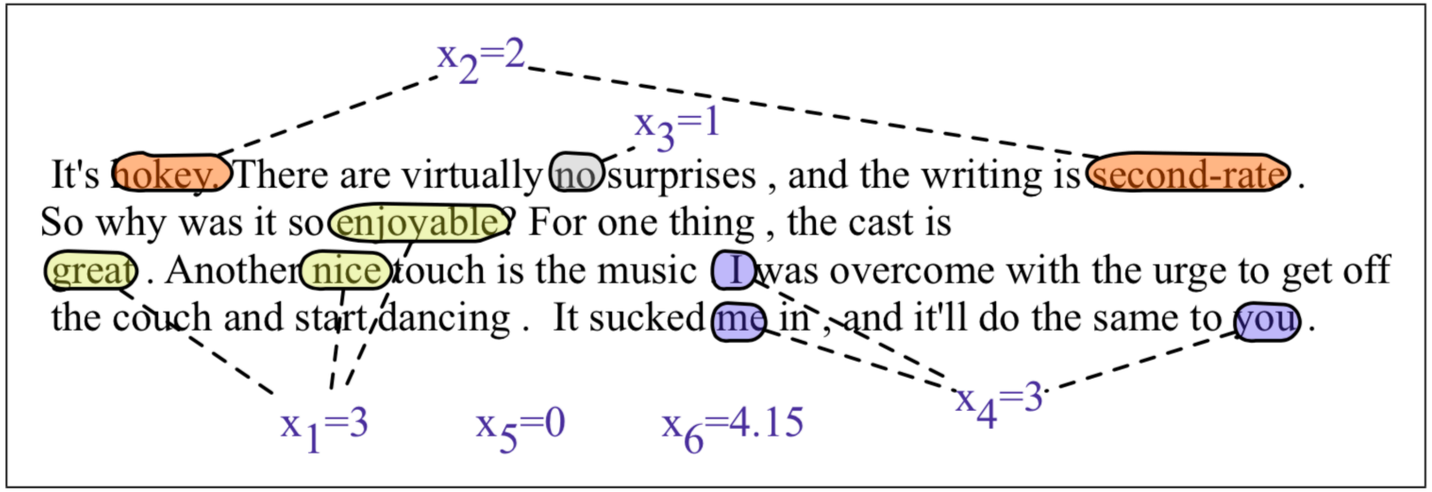
$$= 1 - \frac{1}{1 + e^{-(w \cdot x + b)}} = \frac{e^{-(w \cdot x + b)}}{1 + e^{-(w \cdot x + b)}}$$

- Decision boundary:

$$\hat{y} = \begin{cases} 1 & \text{if } P(y = 1 \mid x) > 0.5 \\ 0 & \text{otherwise} \end{cases}$$

Example: Sentiment classification



Var	Definition	Value in Fig. 5.2
x_1	count(positive lexicon) $\in$ doc)	3
x_2	count(negative lexicon) $\in$ doc)	2
x_3	$\begin{cases} 1 & \text{if ``no'' } \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	1
x_4	count(1st and 2nd pronouns $\in$ doc)	3
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&= \sigma([2.5, -5.0, -1.2, 0.5, 2.0, 0.7] \cdot [3, 2, 1, 3, 0, 4.15] + 0.1) \\
&= \sigma(.805) \\
&= 0.69 \\
p(-|x) = P(Y=0|x) &= 1 - \sigma(w \cdot x + b) \\
&= 0.31
\end{aligned}
$$

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- Advanced: Representation learning (we will see this later!)

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    - $\max \log P(w_t \mid w_{t-n}, \ldots, w_{t-1})$ given a corpus

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  - $y = 1 \implies -\log \hat{y}$, and $y = 0 \implies -\log(1 - \hat{y})$

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- **Loss:** $-\log \prod_{i=1}^{n} P(y \mid x) = -\sum_{i=1}^{n} \log P(y \mid x)$

$$L_{CE} = -\sum_{i=1}^{n} [y \log \hat{y} + (1-y)\log(1-\hat{y})]$$

Example: Computing CE Loss

Var	Definition	Value
x_1	count(positive lexicon) $\in$ doc)	3
x_2	count(negative lexicon) $\in$ doc)	2
x_3	$\begin{cases} 1 & \text{if ``no'' } \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	1
x_4	count(1st and 2nd pronouns $\in$ doc)	3
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x_6	log(word count of doc)	$\ln(64) = 4.15$

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- Assume weights $w = [2.5, -5.0, -1.2, 0.5, 2.0, 0.7]$ and bias $b = 0.1$
- If y = 1 (positive sentiment), $L_{CE} = -\log(0.69) = 0.37$
- If y = 0 (negative sentiment), $L_{CE} = -\log(0.31) = 1.17$

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- *Cross-entropy* between the true distribution $P(y \mid x)$ and predicted distribution $P(\hat{y} \mid x)$

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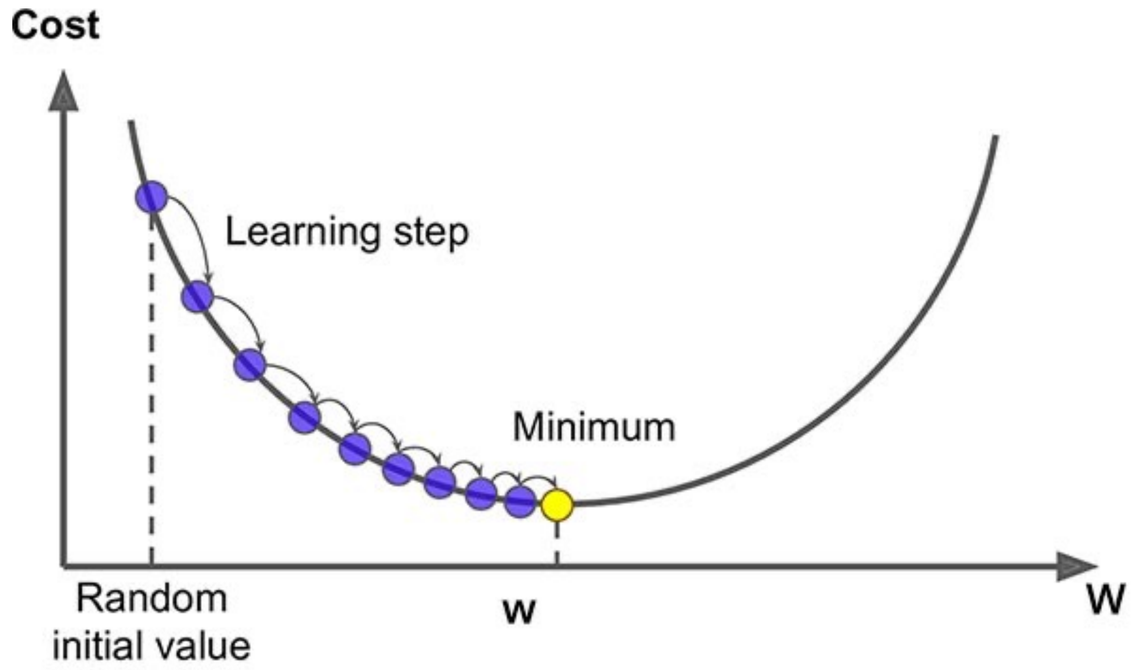
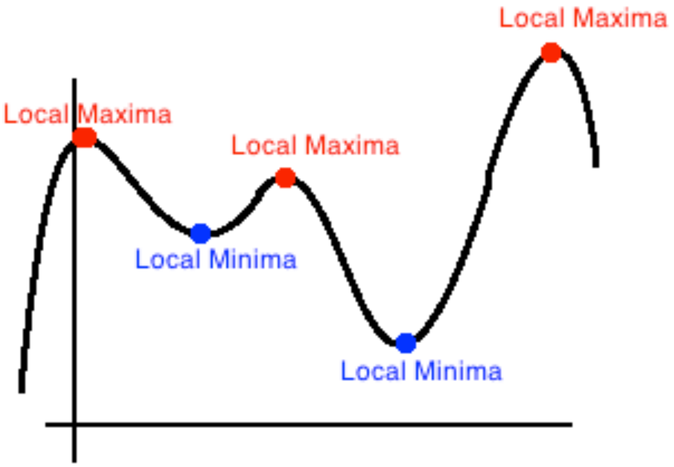
Gradient descent (1-D)



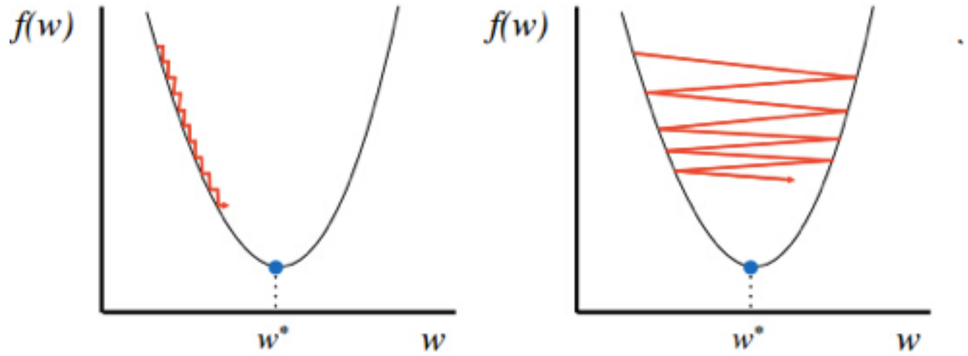
$$\theta^{t+1} = \theta^t - \eta \frac{d}{d\theta} f(x;\theta)$$

Gradient descent for LR

- Cross entropy loss for logistic regression is **convex** (i.e. has only one global minimum)
  - No local minima to get stuck in
- Deep neural networks are not so easy
  - Non-convex



Learning Rate



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Too big: overshoot and even diverge

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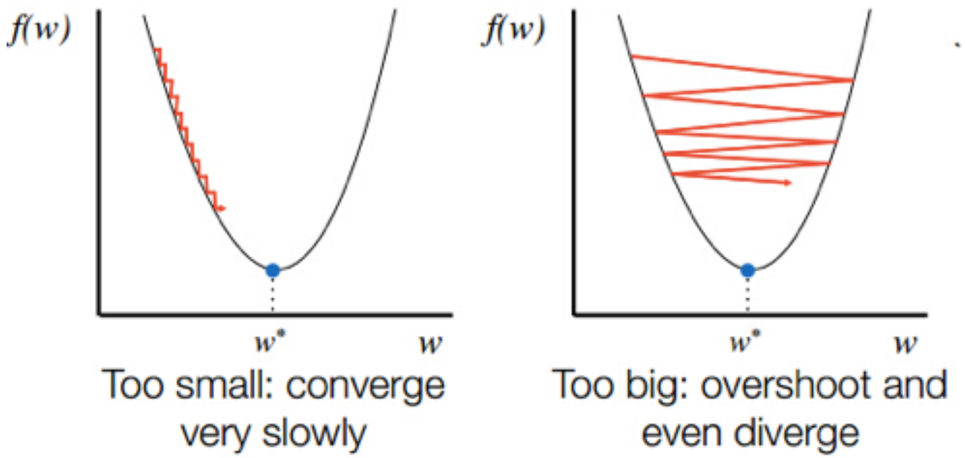

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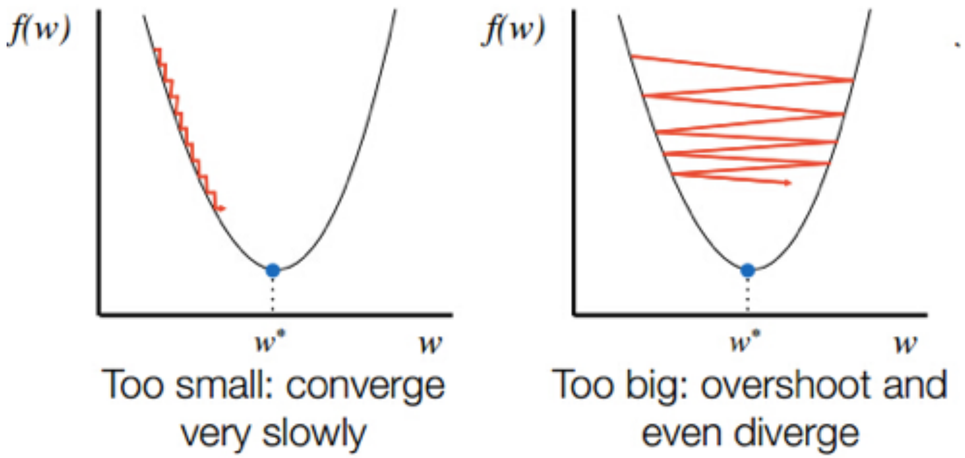

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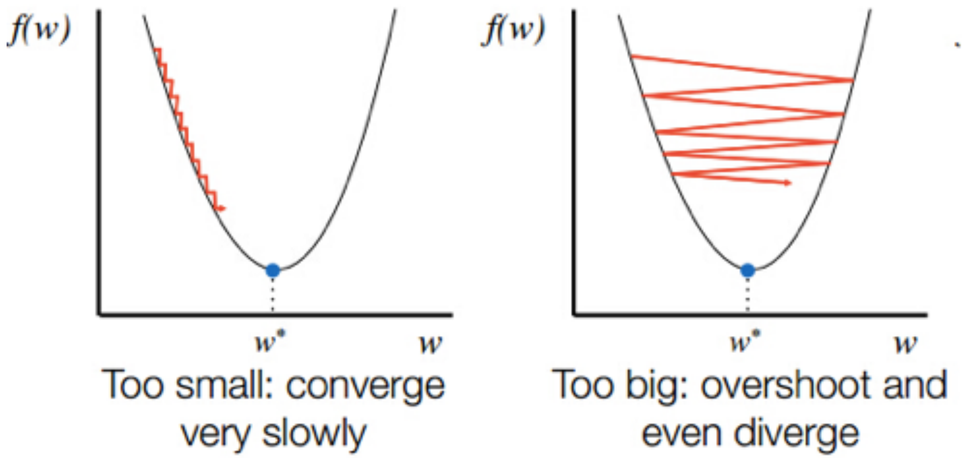

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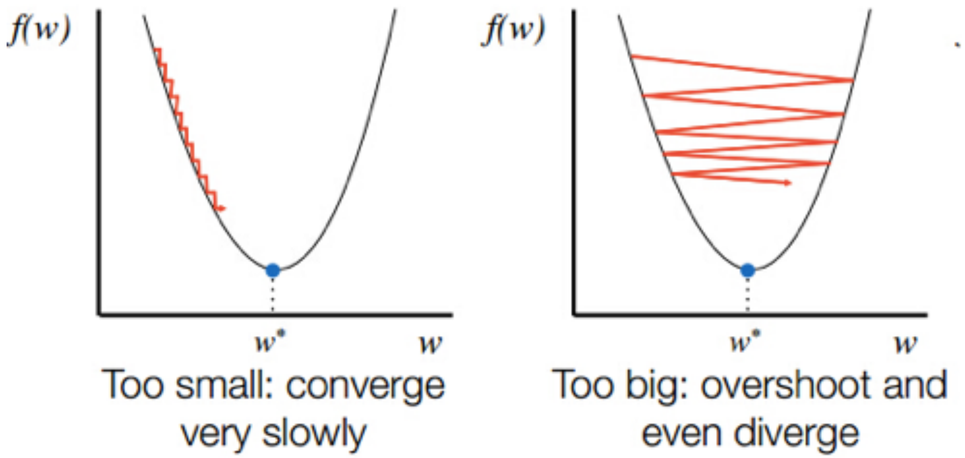

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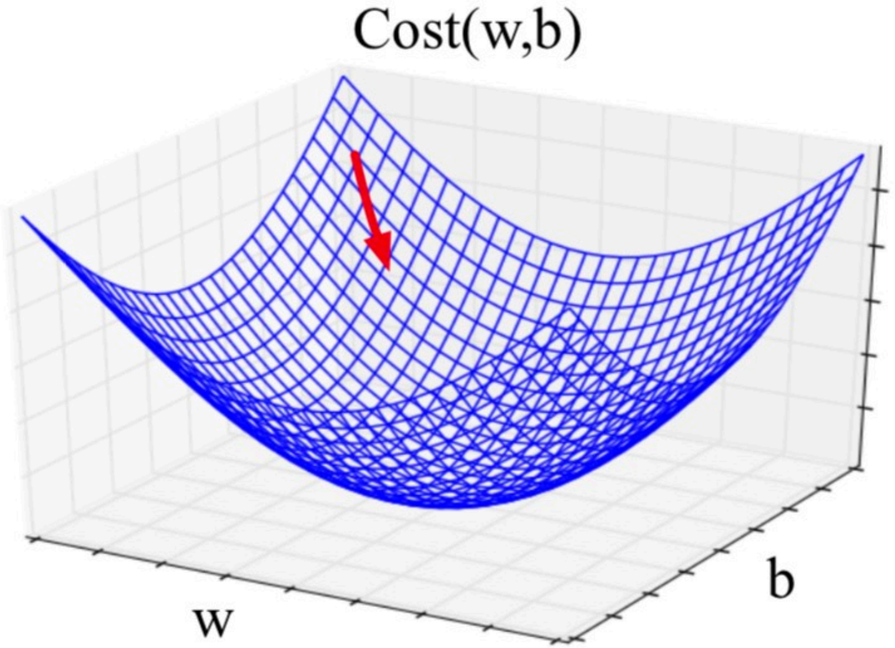
Gradient descent with vector weights



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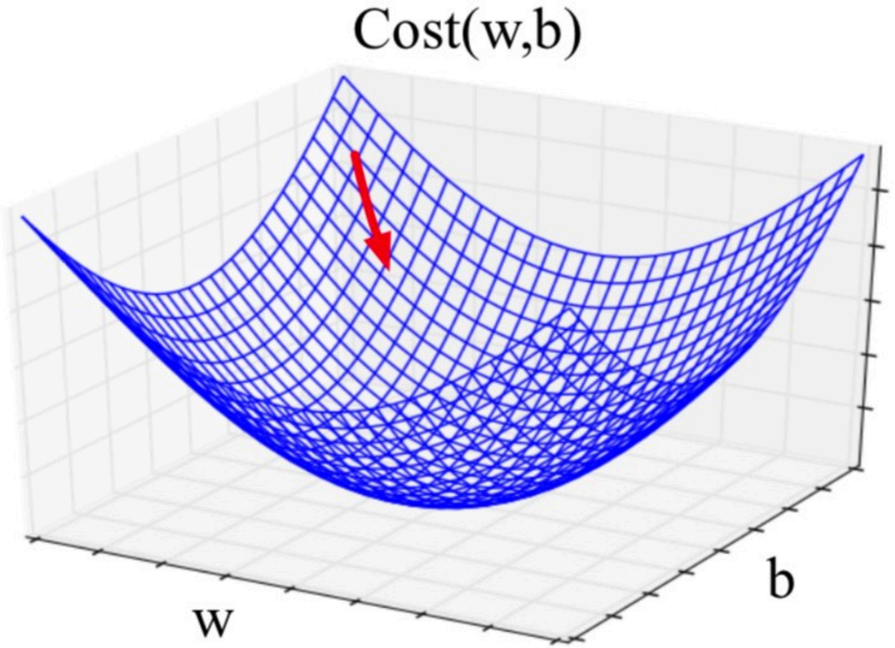

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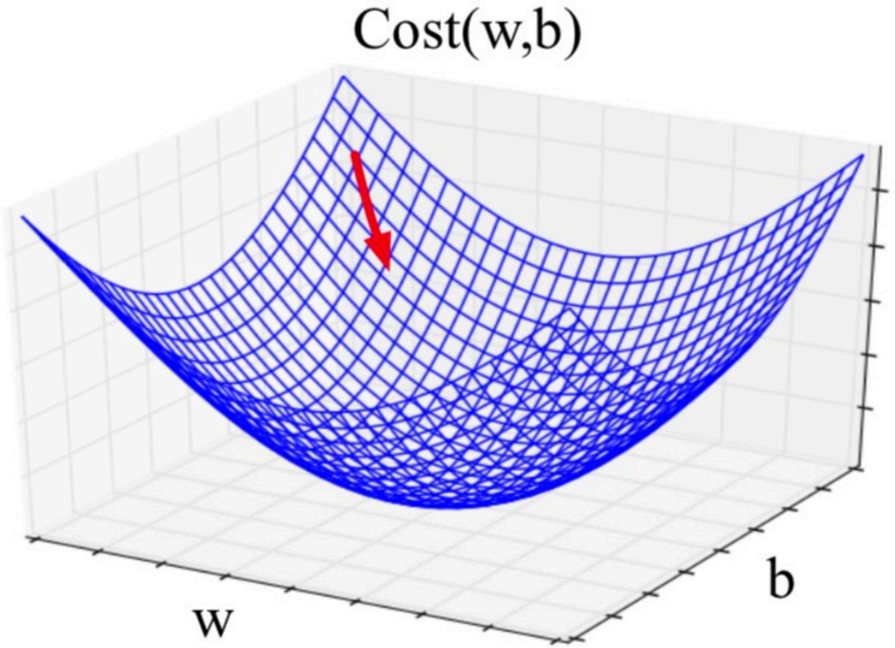

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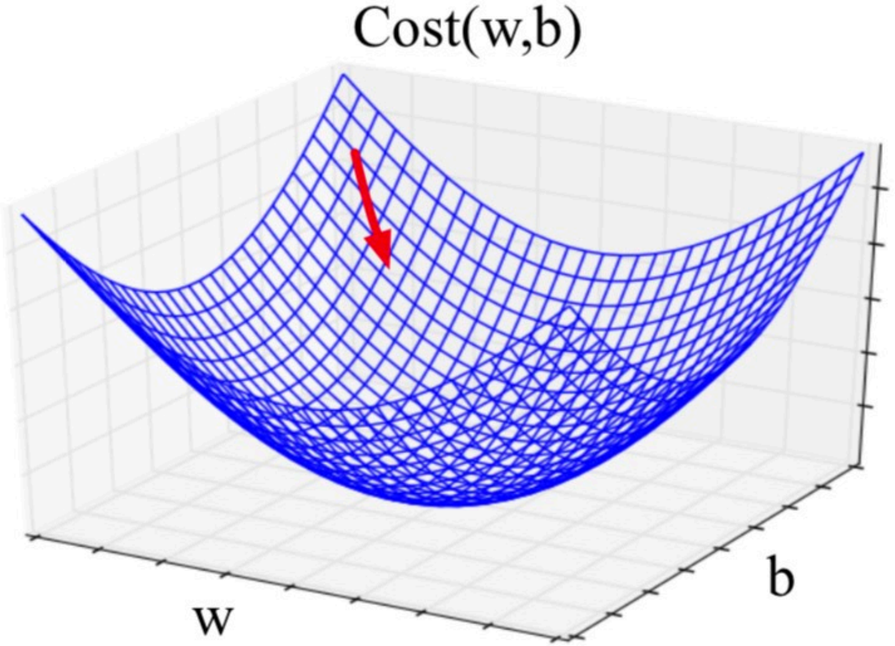

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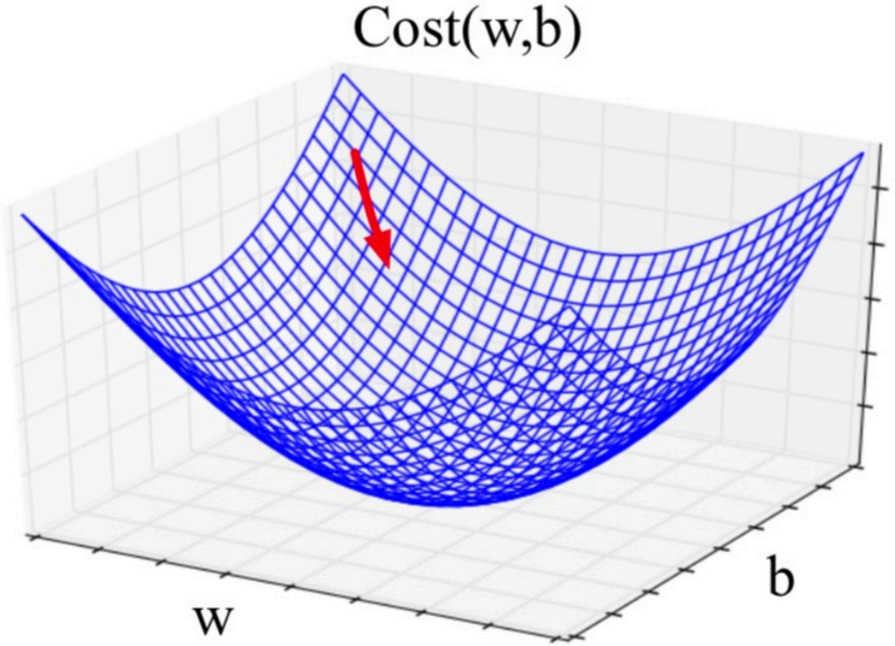

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Stochastic Gradient Descent

- Online optimization
- Compute loss and minimize after **each** training example

function STOCHASTIC GRADIENT DESCENT($L()$, $f()$, x, y) **returns** θ

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    # where: L is the loss function
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θ ← 0
repeat til done    # see caption
  For each training tuple (x^(i), y^(i)) (in random order)
    1. Optional (for reporting):          # How are we doing on this tuple?
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Per Instance Loss

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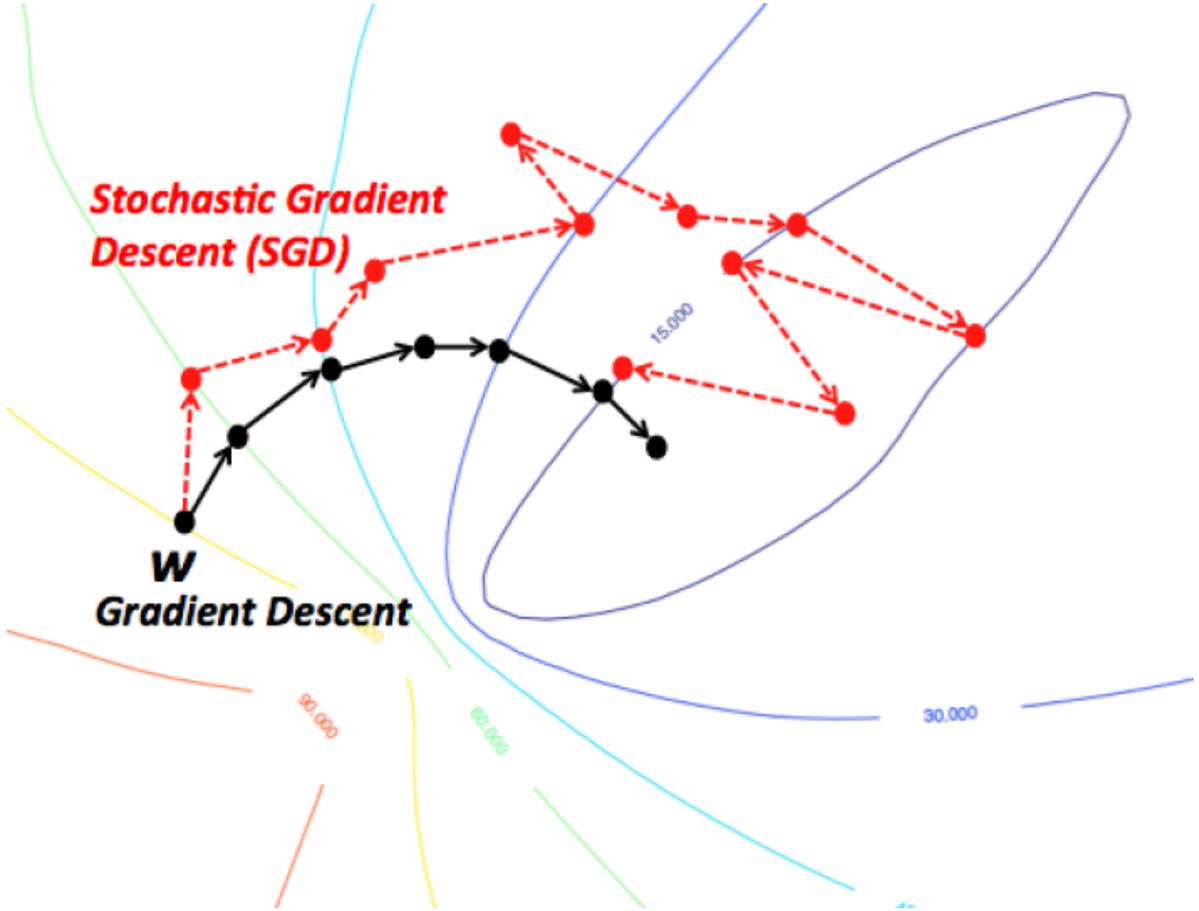

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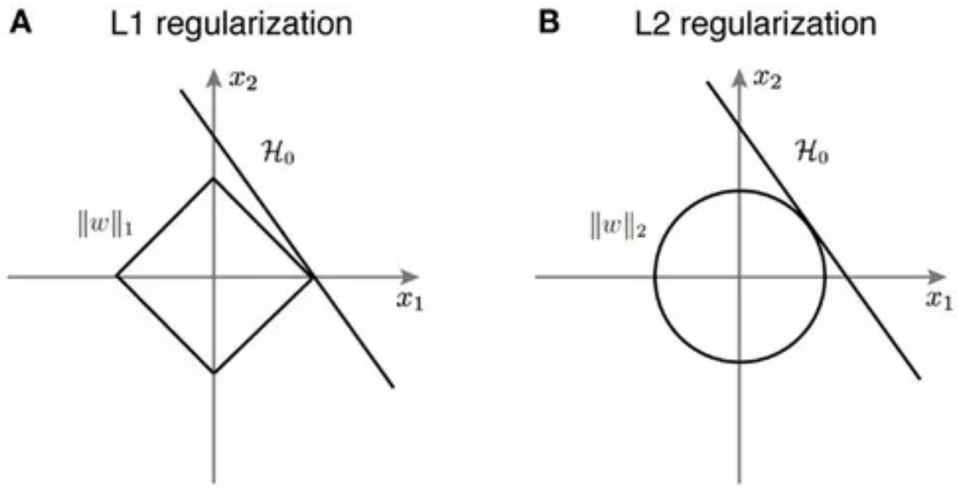

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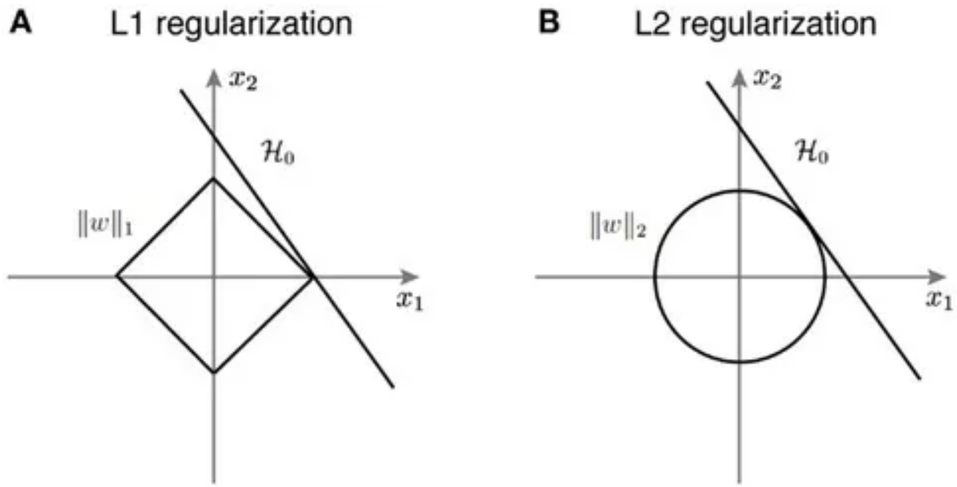

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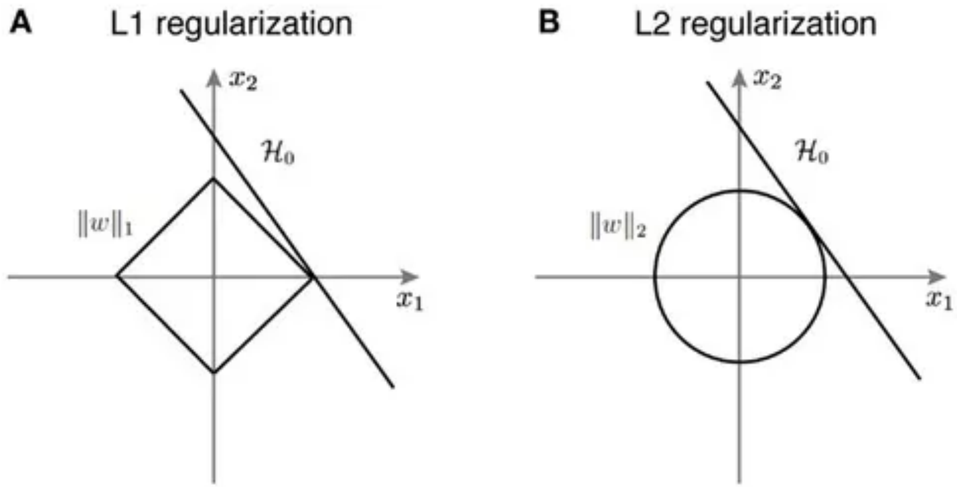

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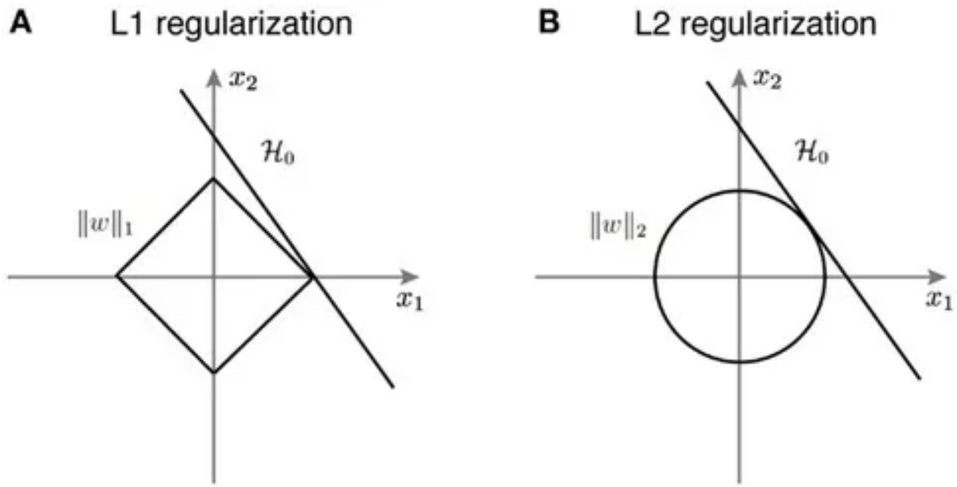

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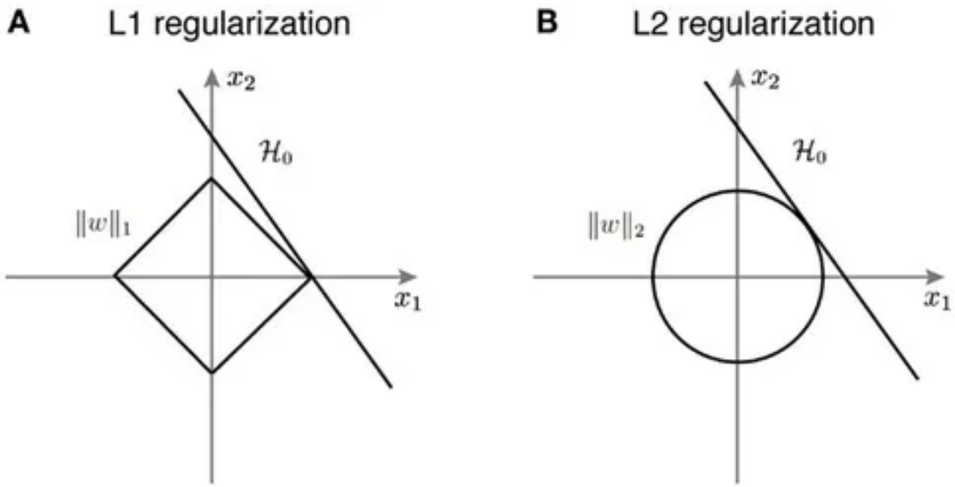

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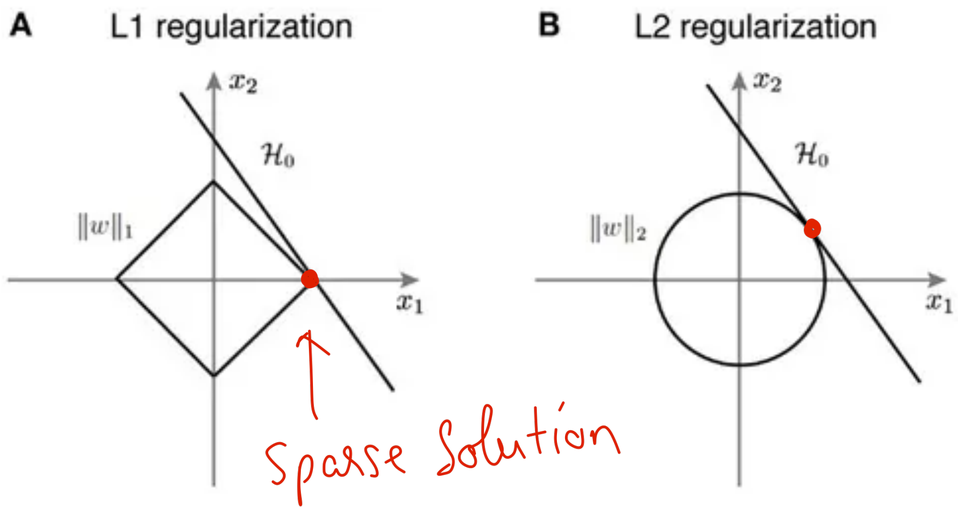

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Softmax

- Similar to sigmoid, softmax squashes values towards 0 or 1
- If $z = [0,1,2,3,4]$, then
  - softmax$(z) = ([0.0117,0.0317,0.0861,0.2341,0.6364])$
- For multinomial LR,

$$P(y = c \mid x) = \frac{e^{w_c \cdot x + b_c}}{\sum_{j=1}^{k} e^{w_j \cdot x + b_j}}$$

Features in multinomial LR

- Features need to include both input (x) and class (c)

- Implicit in binary case

Var	Definition	Wt
$f_1(0,x)$	$\begin{cases} 1 & \text{if } \text{“!”} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	-4.5
$f_1(+,x)$	$\begin{cases} 1 & \text{if } \text{“!”} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	2.6
$f_1(-,x)$	$\begin{cases} 1 & \text{if } \text{“!”} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	1.3

Learning

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- Generalize binary loss to multinomial CE loss:

$$L_{CE}(\hat{y}, y) = -\sum_{c=1}^{k} 1\{y = c\} \log P(y = c \mid x)$$

$$= -\sum_{c=1}^{k} 1\{y = c\} \log \frac{e^{w_c \cdot x + b_c}}{\sum_{j=1}^{k} e^{w_j \cdot x + b_j}}$$

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Binary CE **Loss:**

$$-\log P(y \mid x) = -\left[y \log \hat{y} + (1 - y) \log(1 - \hat{y})\right]$$

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- Gradient:

$$\frac{dL_{CE}}{dw_c} = -(1\{y = c\} - P(y = c \mid x))x$$

$$= -\left(1\{y = c\} - \frac{e^{w_c \cdot x + b_c}}{\sum_{j=1}^{k} e^{w_j \cdot x + b_j}}\right)x$$

Give us feedback!

https://forms.gle/XJXbFLsJCfSNsjCUA