COS 484

Natural Language Processing

L4: Word Embeddings (1)

Spring 2022

(Some slides adapted from Chris Manning, Dan Jurafsky)

How should we represent the meaning of a word?

Recap

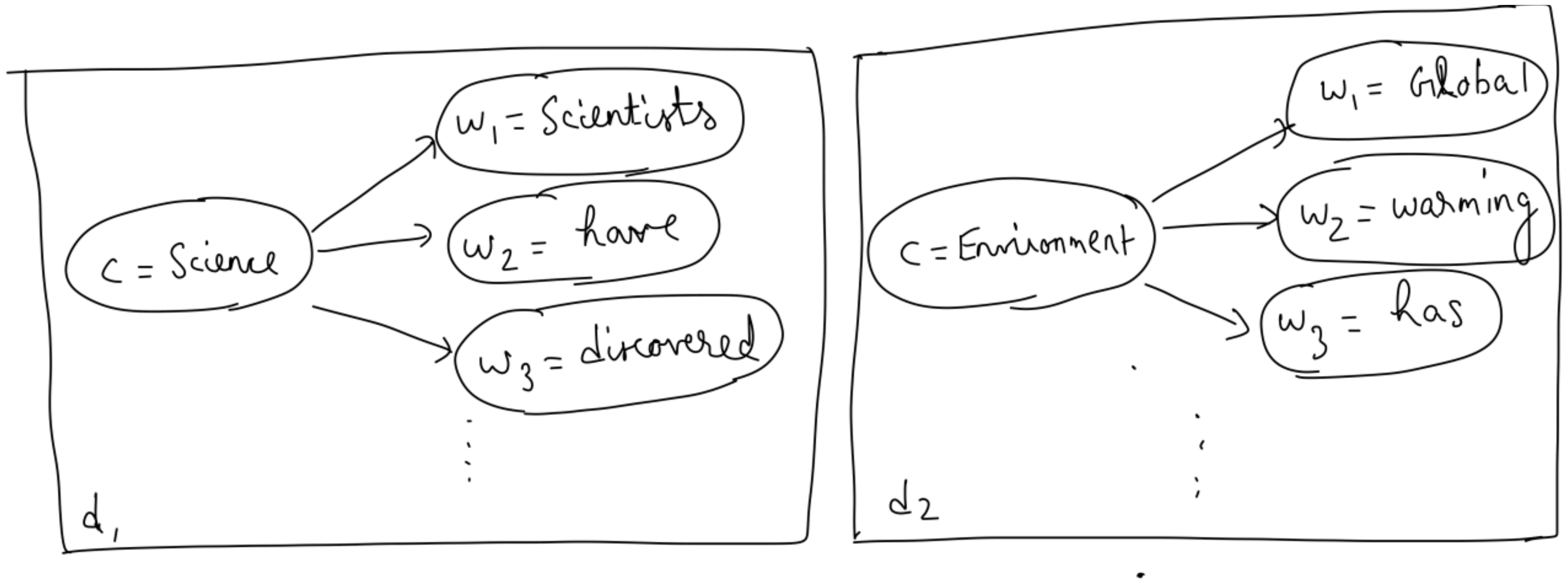
- n-gram models

$$P(\text{the cat sat on the mat}) \approx P(\text{the} \mid \text{START}) \times P(\text{cat} \mid \text{the}) \times P(\text{sat} \mid \text{cat}) \times P(\text{on} \mid \text{sat}) \times P(\text{the} \mid \text{on}) \times P(\text{mat} \mid \text{the})$$

$$P(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i) + \alpha}{C(w_{i-1}) + \alpha|V|}$$

- Naive Bayes

$$\hat{P}(w_i \mid c_j) = \frac{\text{Count}(w_i, c_j) + \alpha}{\sum_{w \in V} \left[\text{Count}(w, c_j) + \alpha\right]}$$



Recap

- Logistic regression

Whether the word “no” appears in the document or not

Var	Definition	Value
x_1	count(positive lexicon) $\in$ doc)	3
x_2	count(negative lexicon) $\in$ doc)	2
x_3	$\begin{cases} 1 & \text{if "no"} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	1
x_4	count(1st and 2nd pronouns $\in$ doc)	3
x_5	$\begin{cases} 1 & \text{if "!"} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	0
x_6	log(word count of doc)	$\ln(64) = 4.15$

Q: How are words represented in these models?

Representing words as discrete symbols

Traditional NLP often regarded words as discrete symbols:

hotel, conference, motel — a localist representation

one 1, the rest 0's

Words can be represented by one-hot vectors:

hotel = [0 0 0 0 0 0 0 0 0 0 1 0 0 0 0]
motel = [0 0 0 1 0 0 0 0 0 0 0 0 0 0 0]

Vector dimension = number of words in vocabulary (e.g., 500,000)

Q: Why is this representation not good?

Why is this representation not good?

If we use **word identity** as features,

⟹ it requires **exact same** word to be in training and test

Training hotel = [0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0]

Test motel = [0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0]

If we use **word vectors** as features,

⟹ We can generalize to **similar** but **unseen** words at testing time!!!

Training hotel = [35, 22, 17, ..]

Test motel = [34, 21, 14, ..]

How do we know the meaning of a word?

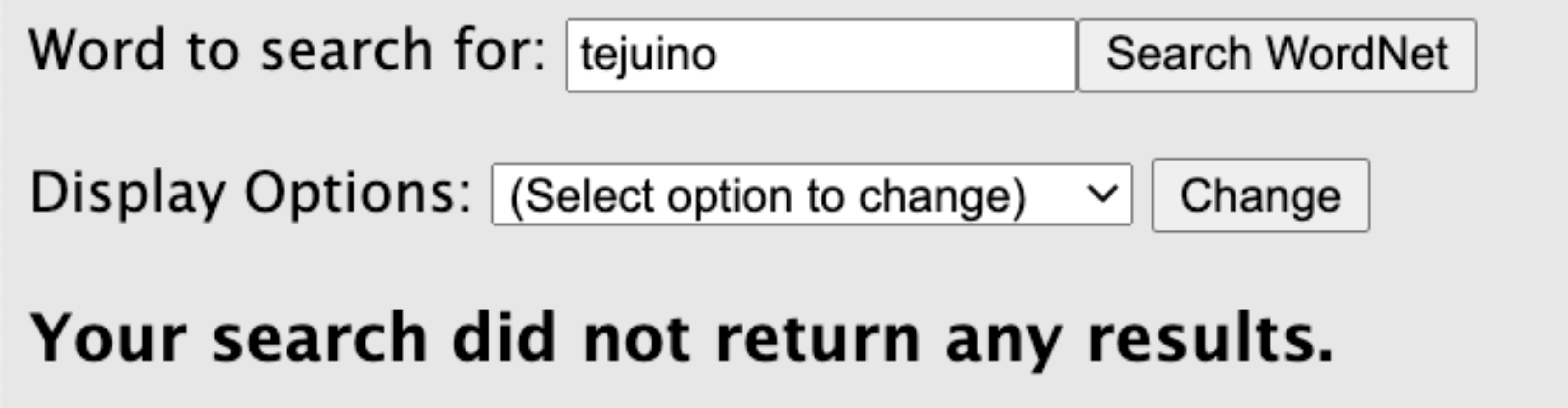
- You can look up the word in a dictionary/thesaurus!

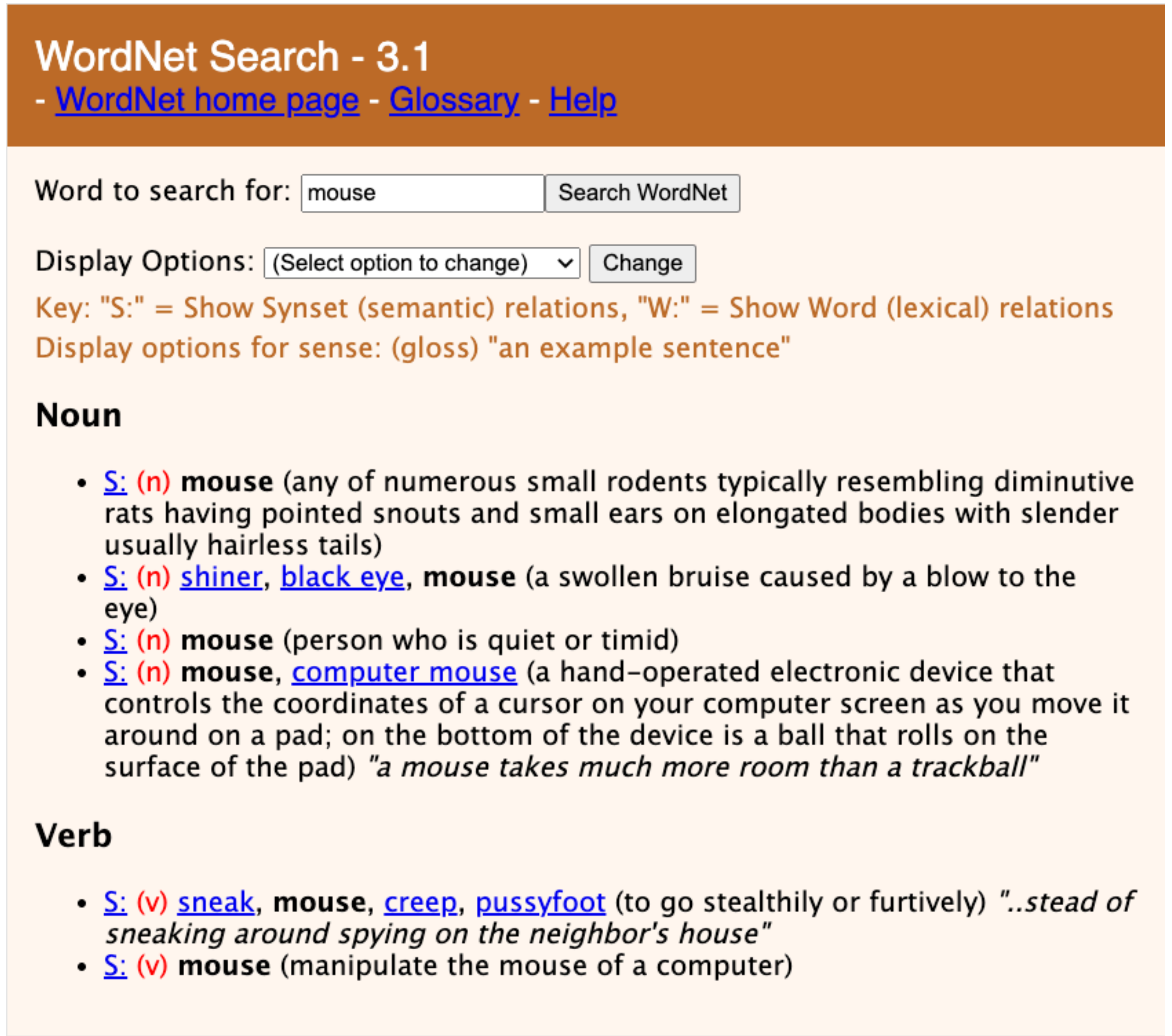
"Princeton"

1. a university town in central New Jersey
2. a university in New Jersey

The meaning of words can be defined by other words!

"tejuino"





http://wordnetweb.princeton.edu/

Key idea: you can know the meaning of a word by looking at its context words

Representing words by their context

Distributional hypothesis: words that occur in similar contexts tend to have similar meanings

J.R.Firth 1957

- "You shall know a word by the company it keeps"
- One of the most successful ideas of modern statistical NLP!

When a word *w* appears in a text, its context is the set of words that appear nearby (within a fixed-size window).

…government debt problems turning into	***banking***	*crises as happened in 2009…*
…saying that Europe needs unified	***banking***	*regulation to replace the hodgepodge…*
…India has just given its	***banking***	*system a shot in the arm…*

These context words will represent "*banking*".

Distributional hypothesis

"tejuino"

C1: A bottle of ___ is on the table.

C2: Everybody likes ___.

C3: Don't have ___ before you drive.

C4: We make ___ out of corn.

Q: What do you think 'tejuino' means?

A) a savory snack
B) a type of flour
C) an alcoholic beverage
D) a type of dessert

Distributional hypothesis

"tejuino"

C1: A bottle of ___ is on the table.

C2: Everybody likes ___.

C3: Don't have ___ before you drive.

C4: We make ___ out of corn.

Tejuíno is a cold beverage made from fermented corn and popularly consumed in the Mexican states of Jalisco and Chihuahua.

Distributional hypothesis

C1: A bottle of ____ is on the table.

C2: Everybody likes ____.

C3: Don't have ____ before you drive.

C4: We make ____ out of corn.

	C1	C2	C3	C4
tejuino	1	1	1	1
loud	0	0	0	0
motor-oil	1	0	0	0
tortillas	0	1	0	1
choices	0	1	0	0
wine	1	1	1	0

Q: Which word is closest to "tejuino"?

"words that occur in similar contexts tend to have similar meanings"

Words as vectors

Q: What is the dimension of each such vector?

Let's build a new model of meaning focusing on similarity

- Each word is a vector
- Similar words are "nearby in space"

A first solution: we can just use **word-word co-occurrence counts** to represent the meaning of words!

context words:
4 words to the left +
4 words to the right

is traditionally followed by	**cherry**	pie, a traditional dessert
often mixed, such as	**strawberry**	rhubarb pie. Apple pie
computer peripherals and personal	**digital**	assistants. These devices usually
a computer. This includes	**information**	available on the internet

	aardvark	...	computer	data	result	pie	sugar	...
cherry	0	...	2	8	9	442	25	...
strawberry	0	...	0	0	1	60	19	...
digital	0	...	1670	1683	85	5	4	...
information	0	...	3325	3982	378	5	13	...

Most entries are 0s $\Longrightarrow$ sparse vectors

Words as vectors

context words: 4 words to the left, 4 words to the right

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vs.

C1: A bottle of ____ is on the table.

C2: Everybody likes ____.

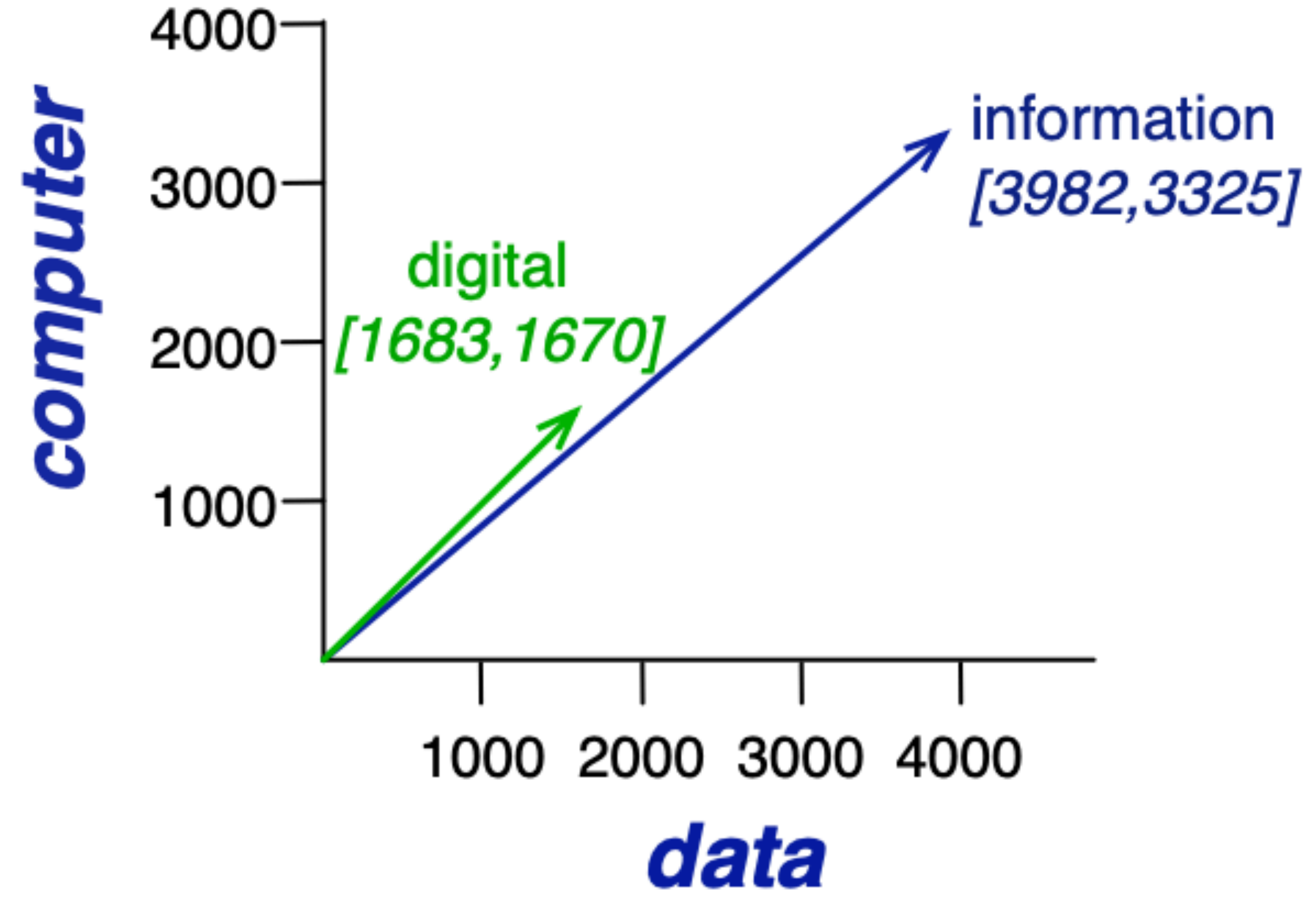
C3: Don't have ____ before you drive.

C4: We make ____ out of corn.

	C1	C2	C3	C4
tejuino	1	1	1	1
loud	0	0	0	0
motor-oil	1	0	0	0
tortillas	0	1	0	1
choices	0	1	0	0
wine	1	1	1	0

Using C_i is too sparse.
Word-word co-occurrence can be thought of as a simplification + frequency captures important information!

Measuring similarity



A common similarity metric: **cosine** of the angle between the two vectors (the larger, the more similar the two vectors are)

$$\cos(\mathbf{u}, \mathbf{v}) = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|}$$

$$\cos(\mathbf{u}, \mathbf{v}) = \frac{\sum_{i=1}^{|V|} u_i v_i}{\sqrt{\sum_{i=1}^{|V|} u_i^2} \sqrt{\sum_{i=1}^{|V|} v_i^2}}$$

Q: Why cosine similarity instead of dot product $\mathbf{u} \cdot \mathbf{v}$?

Quick poll

What is the range of $\cos(u, v)$ if u, v are count vectors?

(a) [-1, 1]

(b) [0, 1]

(c) $[0, +\infty)$

(d) $(-\infty, +\infty)$

$$\cos(\mathbf{u}, \mathbf{v}) = \frac{\sum_{i=1}^{|V|} u_i v_i}{\sqrt{\sum_{i=1}^{|V|} u_i^2}\sqrt{\sum_{i=1}^{|V|} v_i^2}}$$

The answer is (b). Cosine similarity ranges between -1 and 1 in general. In this model, all the values of u_i, v_i are non-negative.

Any issues with this model?

Raw frequency count is a bad representation!

- Frequency is clearly useful; if “pie” appears a lot near “cherry”, that's useful information.
- But overly frequent words like “the”, “it", or “they” also appear a lot near “cherry”. They are not very informative about the context.

Solution: use a **weighted function** instead of raw counts!

Pointwise Mutual Information (PMI):

Do events *x* and *y* co-occur more or less than if they were independent?

$$\text{PMI}(x, y) = \log_2 \frac{P(x, y)}{P(x)P(y)} \qquad \text{PMI}(\text{word}_1, \text{word}_2) = \log_2 \frac{P(\text{word}_1, \text{word}_2)}{P(\text{word}_1)P(\text{word}_2)}$$

Positive Pointwise Mutual Information (PPMI)

- PMI ranges from $-\infty$ to $+\infty$
- $\mathrm{PMI}(w_1, w_2) > 0 \implies P(w_1, w_2) > P(w_1)P(w_2)$
- $\mathrm{PMI}(w_1, w_2) < 0 \implies P(w_1, w_2) < P(w_1)P(w_2)$
- When one or both words are rare, there is high sampling error in their probabilities
- Negative values of PMI are frequently not reliable
- A simple fix: replace all the negative PMI values by 0s

$$\mathrm{PPMI}(\mathrm{word}_1, \mathrm{word}_2) = \max\left(\log_2 \frac{P(\mathrm{word}_1, \mathrm{word}_2)}{P(\mathrm{word}_1)P(\mathrm{word}_2)}, 0\right)$$

Warning: negative PMI values may be statistically significant, and informative in practice, if both words are quite common.

PPMI - A running example

$$p_{ij} = \frac{f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}}$$

	computer	data	result	pie	sugar	count(w)
cherry	2	8	9	442	25	486
strawberry	0	0	1	60	19	80
digital	1670	1683	85	5	4	3447
information	3325	3982	378	5	13	7703
count(context)	4997	5673	473	512	61	11716

p(w=information,c=data) = 3982/111716 = .3399

p(w=information) = 7703/11716 = .6575

p(c=data) = 5673/11716 = .4842

$$p(w_i) = \frac{\sum_{j=1}^{C} f_{ij}}{N} \qquad p(c_j) = \frac{\sum_{i=1}^{W} f_{ij}}{N}$$

	p(w,context)					p(w)
	computer	**data**	**result**	**pie**	**sugar**	**p(w)**
cherry	0.0002	0.0007	0.0008	0.0377	0.0021	0.0415
strawberry	0.0000	0.0000	0.0001	0.0051	0.0016	0.0068
digital	0.1425	0.1436	0.0073	0.0004	0.0003	0.2942
information	0.2838	0.3399	0.0323	0.0004	0.0011	0.6575
p(context)	0.4265	0.4842	0.0404	0.0437	0.0052	

Poll

p(w=information,c=data) = 3982/111716 = .3399
p(w=information) = 7703/11716 = .6575
p(c=data) = 5673/11716 = .4842

$$p(w_i) = \frac{\sum_{j=1}^{C} f_{ij}}{N} \qquad p(c_j) = \frac{\sum_{i=1}^{W} f_{ij}}{N}$$

Assume that we have a text corpus of 1M tokens, we use 4 words before and 4 words after as context **c** for each word **w**, what is N (the denominator for computing these probabilities) approximately?

(a) 1M
(b) 4M
(c) 8M
(d) not enough information

The answer is (c). For every word w_i in the corpus, we need to collect 8 pairs (w_i, w_{i+j}), for j = -4, -3, -2, -1, 1, 2, 3, 4.

PPMI - A running example

	p(w,context)					p(w)
	computer	data	result	pie	sugar	p(w)
cherry	0.0002	0.0007	0.0008	0.0377	0.0021	0.0415
strawberry	0.0000	0.0000	0.0001	0.0051	0.0016	0.0068
digital	0.1425	0.1436	0.0073	0.0004	0.0003	0.2942
information	0.2838	0.3399	0.0323	0.0004	0.0011	0.6575
p(context)	0.4265	0.4842	0.0404	0.0437	0.0052	

$$\text{PMI}(\text{cherry}, \text{pie}) = log_2(0.0377/0.0415/0.0437) = 4.38$$

$$\text{PMI}(\text{cherry}, \text{result}) = log_2(0.0008/0.0415/0.0404) = -1.07$$

$$\text{PMI}(\text{digital}, \text{result}) = log_2(0.0073/0.2942/0.0404) = -0.70$$

Resulting PPMI matrix (negatives replaced by 0)

	computer	data	result	pie	sugar
cherry	0	0	0	4.38	3.30
strawberry	0	0	0	4.10	5.51
digital	0.18	0.01	0	0	0
information	0.02	0.09	0.28	0	0

From sparse vectors to dense vectors

- The vectors in the word-word occurrence matrix are still sparse (most are 0's) & long (vocabulary size)

- Alternative: we want to represent words as **short** (50-300 dimensional) & **dense** (real-valued) vectors
  - The basis for modern NLP systems

$$\text{employees} = \begin{pmatrix} 0.286 \\ 0.792 \\ -0.177 \\ -0.107 \\ 10.109 \\ -0.542 \\ 0.349 \\ 0.271 \\ 0.487 \end{pmatrix}$$

Why dense vectors?

- Short vectors are easier to use as features in ML systems
- Dense vectors generalize better than explicit counts (points in real space vs points in integer space)
- Sparse vectors can't capture higher-order co-occurrence
  - w_1 co-occurs with "car", w_2 co-occurs with "automobile"
  - They should be similar but they aren't because "car" and "automobile" are distinct dimensions
- In practice, they work better!

How to get dense vectors?

Singular value decomposition (SVD) of PPMI weighted co-occurrence matrix

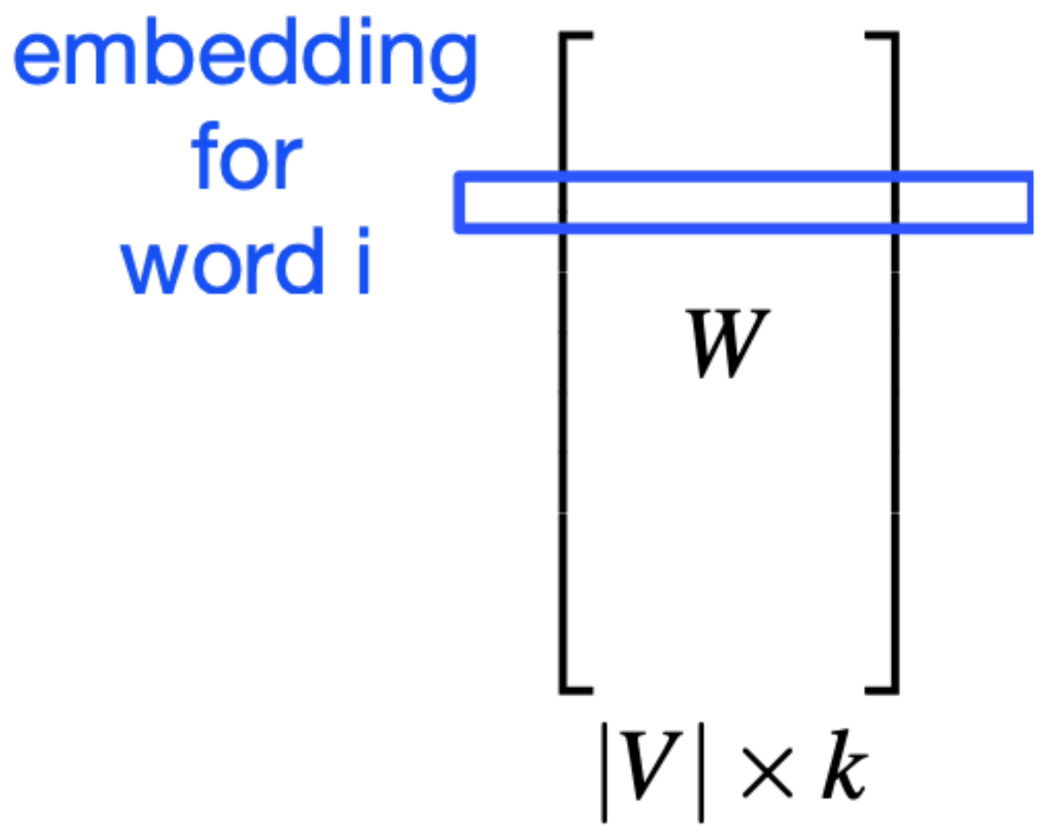
$$\underset{|V| \times |V|}{\begin{bmatrix} \\ \\ & X & \\ \\ \\ \end{bmatrix}} = \underset{|V| \times |V|}{\begin{bmatrix} \\ \\ & W & \\ \\ \\ \end{bmatrix}} \underset{|V| \times |V|}{\begin{bmatrix} \sigma_1 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \sigma_V \end{bmatrix}} \underset{|V| \times |V|}{\begin{bmatrix} \\ \\ & C & \\ \\ \\ \end{bmatrix}}$$

$$\underset{|V| \times |V|}{\begin{bmatrix} \\ \\ & X & \\ \\ \\ \end{bmatrix}} = \underset{|V| \times k}{\begin{bmatrix} \\ \\ W \\ \\ \\ \end{bmatrix}} \underset{k \times k}{\begin{bmatrix} \sigma_1 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \sigma_k \end{bmatrix}} \underset{k \times |V|}{\begin{bmatrix} & C & \end{bmatrix}}$$

Only keep the top k (e.g., 100) singular values!

How to get dense vectors?

- Singular value decomposition (SVD) of PPMI weighted co-occurrence matrix
  - Each row of the matrix W is a k-dimensional vector for each word w
  - This idea originates from Latent Semantic Analysis (Deerwester et al., 1990) (applied on word-document matrix)



- Alternative approach: **learning** word vectors directly from text
  - Popular methods: **word2vec (Mikolov et al., 2013)**, Glove (Pennington et al., 2014), FastText (Bojanowski et al., 2017)
  - Key idea: Instead of counting how often each word w co-occurs with another word v and perform matrix factorization, we use the dense vector of w to predict v (a machine learning problem!)

Count-based vs prediction-based word vectors

- Recommended reading: (Baroni et al., 2014)

Don't count, predict! A systematic comparison of context-counting vs. context-predicting semantic vectors

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These context-predicting word vectors are also called word embeddings...