Precept 2: Classification

COS 484

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2/8 2023

Review Question

You train an n-gram model on some training corpus D using counts $P(w_n \mid w_1, \ldots, w_{n-1}) = \frac{c(w_1, \ldots, w_n)}{\sum_{v \in V} c(w_1, \ldots, w_{n-1}, v)}$. To prevent (possible) infinite perplexity on the test corpus D_t, you apply Laplace smoothing. Let $P(D), P(D_t)$ be the unsmoothed probabilities and $P'(D), P'(D_t)$ be the smoothed probabilities.

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1. $ppl'(D) \geq ppl(D)$

2. $ppl'(D_t) < ppl(D_t)$

3. $ppl(D) < ppl(D_t)$

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1. $ppl'(D) \geq ppl(D)$
   This is true! Remember that setting the probability using counts (above) is the MLE estimate, which means that $P(D)$ cannot increase under any other distribution for $P(w_n \mid w_1, \ldots, w_{n-1})$

2. $ppl'(D_t) < ppl(D_t)$

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1. $ppl'(D) \geq ppl(D)$ - T

2. $ppl'(D_t) < ppl(D_t)$

   This is undetermined! It's not clear that the test corpus will have infinite perplexity. It is possible that the test corpus is very similar to the train corpus, and smoothing will cause its probability to drop.

3. $ppl(D) < ppl(D_t)$

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3. $ppl(D) < ppl(D_t)$
   Undetermined. A test corpus consisting solely of high-frequency n-grams might have a higher probability

Todays Topics

Given a document $d = w_1, \ldots, w_K$ and a set of classes $\mathscr{C} = \{c_1, \ldots, c_m\}$, we want to find the class c_i that maximizes $P(c \mid d)$. Two ways to do this:

Naive Bayes

Logistic Regression

Naive Bayes Intuition

Given a document $d = w_1, \ldots, w_K$ and a set of classes $\mathscr{C} = \{c_1, \ldots, c_m\}$, we want to find the class c_i that maximizes $P(c \mid d)$ (the MAP estimate)

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Language Model: $P(w_1, \ldots, w_K) = P(d)$ gives us a probability for a text sequence

Conditional Language Model: $P(d \mid c)$ gives us probability of a text sequence conditioned on something

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$$c_{\text{MAP}} = \text{argmax}_{c \in C} P(c \mid d)$$

$$= \text{argmax}_{c \in C} \frac{P(d \mid c) P(c)}{P(d)}$$

$$= \text{argmax}_{c \in C} P(d \mid c) P(c)$$

Naive Bayes: An "illustration"

Summary: We want to find the class $c_{MAP} = \arg\max_{c \in C} P(d \mid c)P(c)$

Let's say you work on a group project with a friend, and we want a model that can attribute your writing vs your friend's writing. C = {you, friend}

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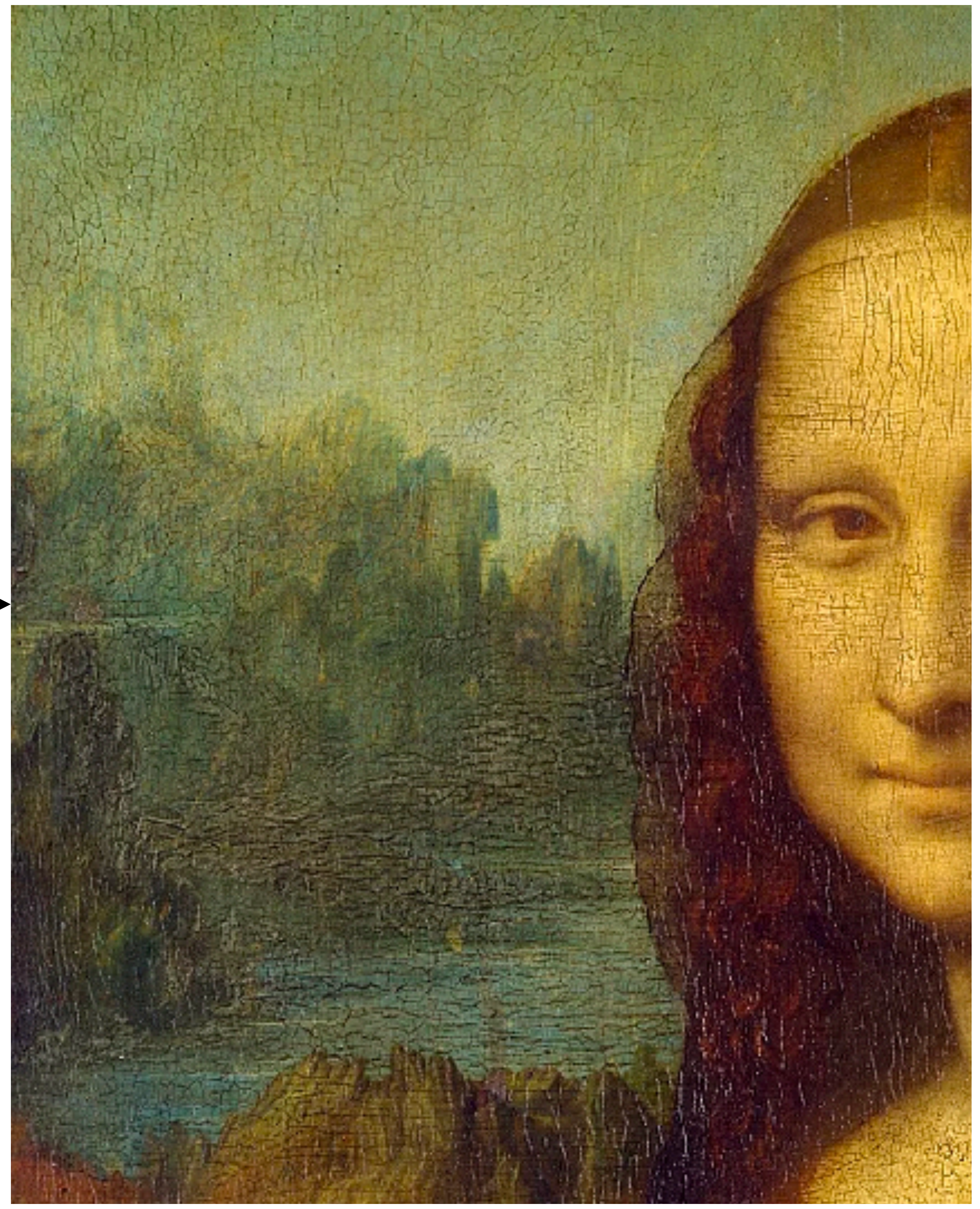
Your friend's writing



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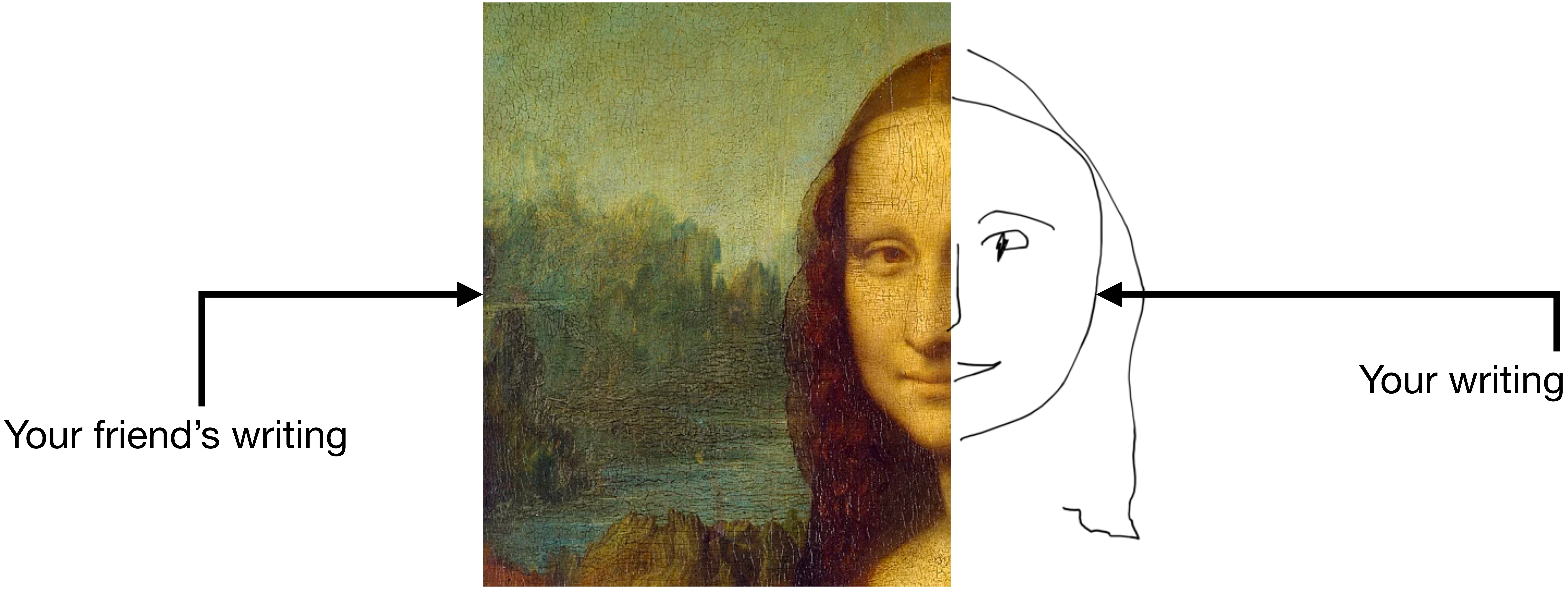

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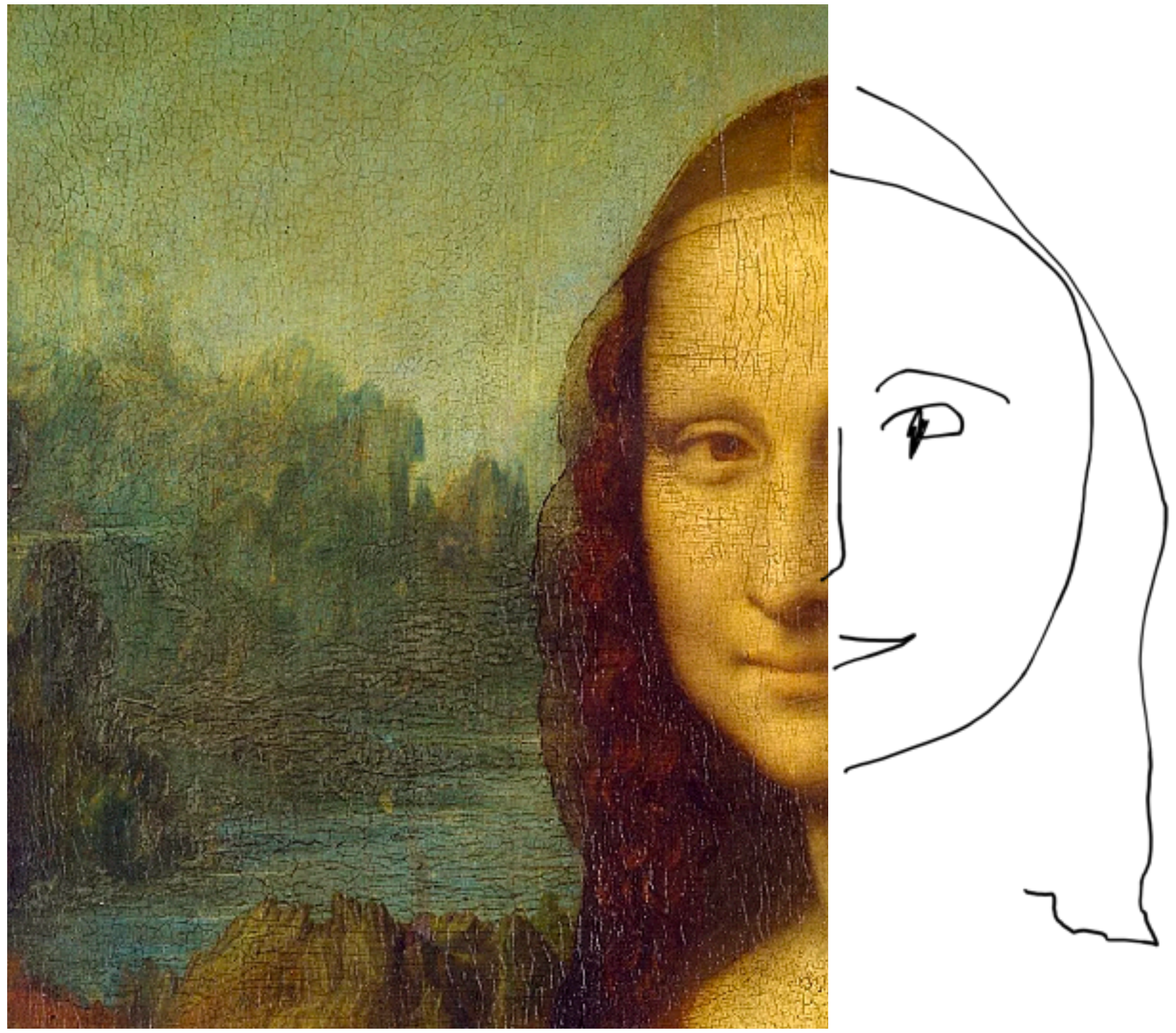
Summary: We want to find the class $c_{MAP} = \arg\max_{c \in C} P(d \mid c) \boxed{P(c)}$

First, we determine who did more work. This gives us a prior estimate (bias) on whether any document was written by you or your friend.



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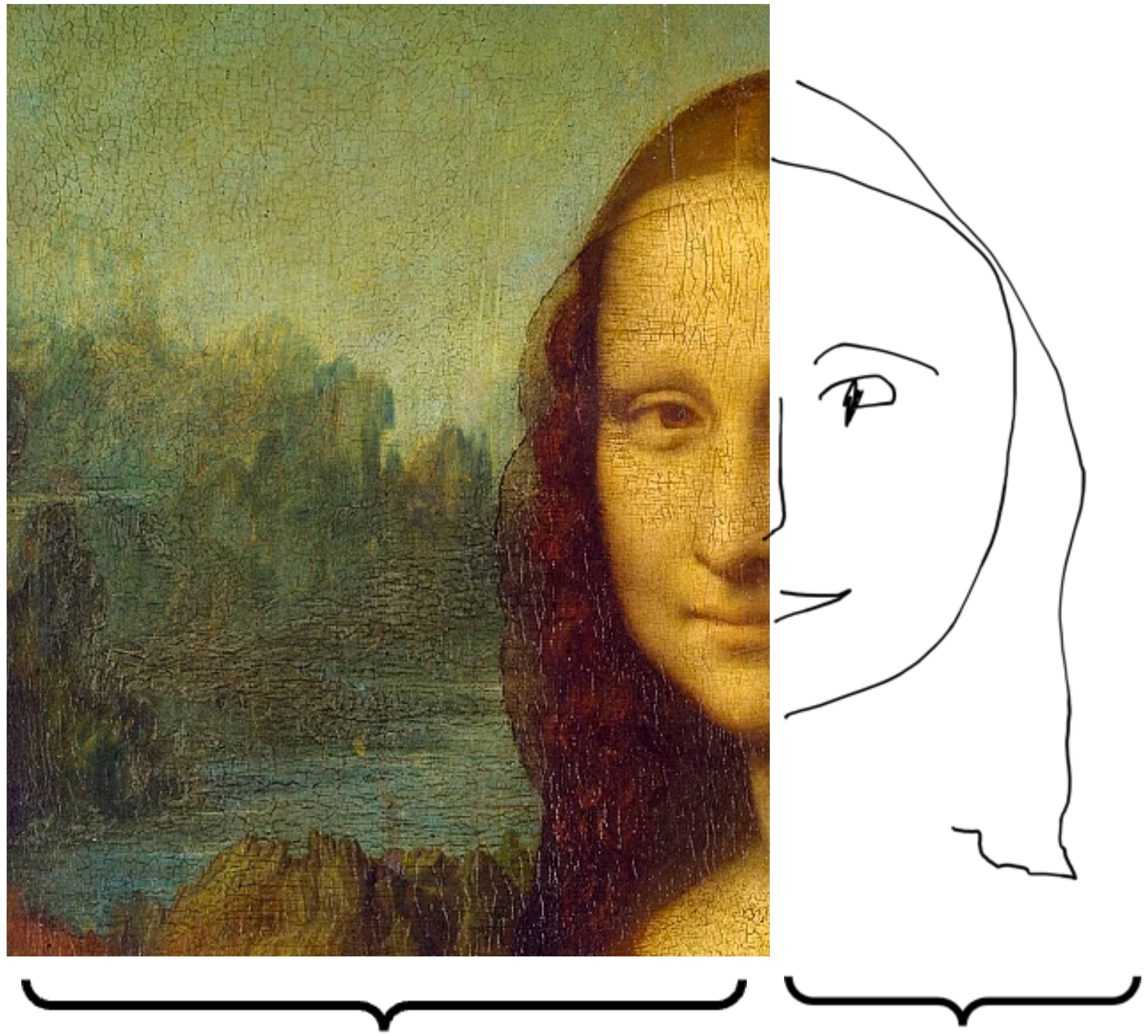

P(friend) = 3/4 P(you) = 1/4

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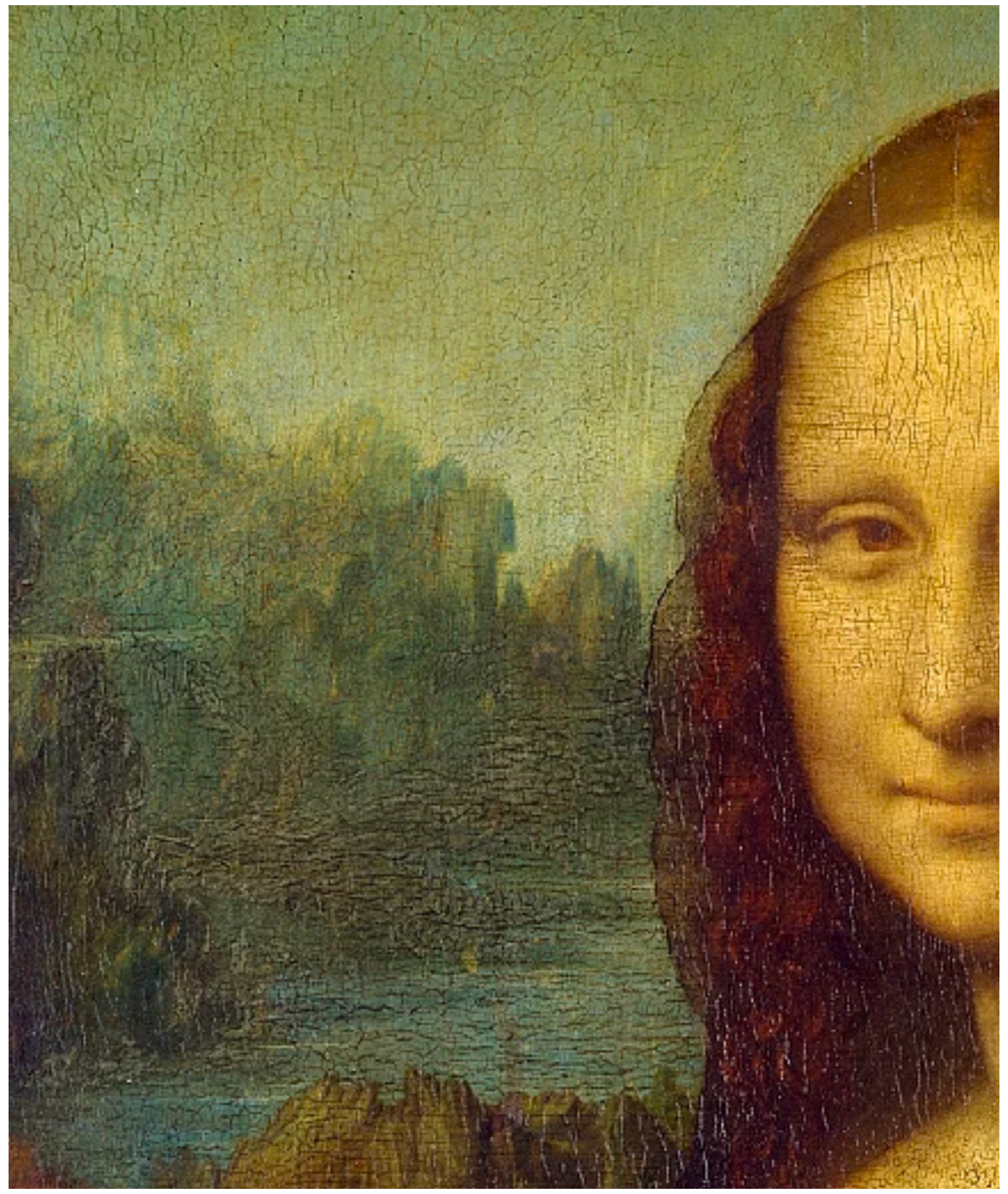
Summary: We want to find the class $c_{MAP} = \underset{c \in C}{\arg\max}\, \boxed{P(d \mid c)} P(c)$

Now, to compute $P(d \mid c)$ for any input document d We can train two language models, one trained on your writing, and one on your friend's



P(x|friend)

P(x|you)

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Summary: We want to find the class $c_{MAP} = \arg\max_{c \in C} P(d \mid c)P(c)$

Now given a new sample d, we can compute the probability under each LM to find $P(d \mid c)$. And multiply this by $P(c)$ to find the MAP estimate.

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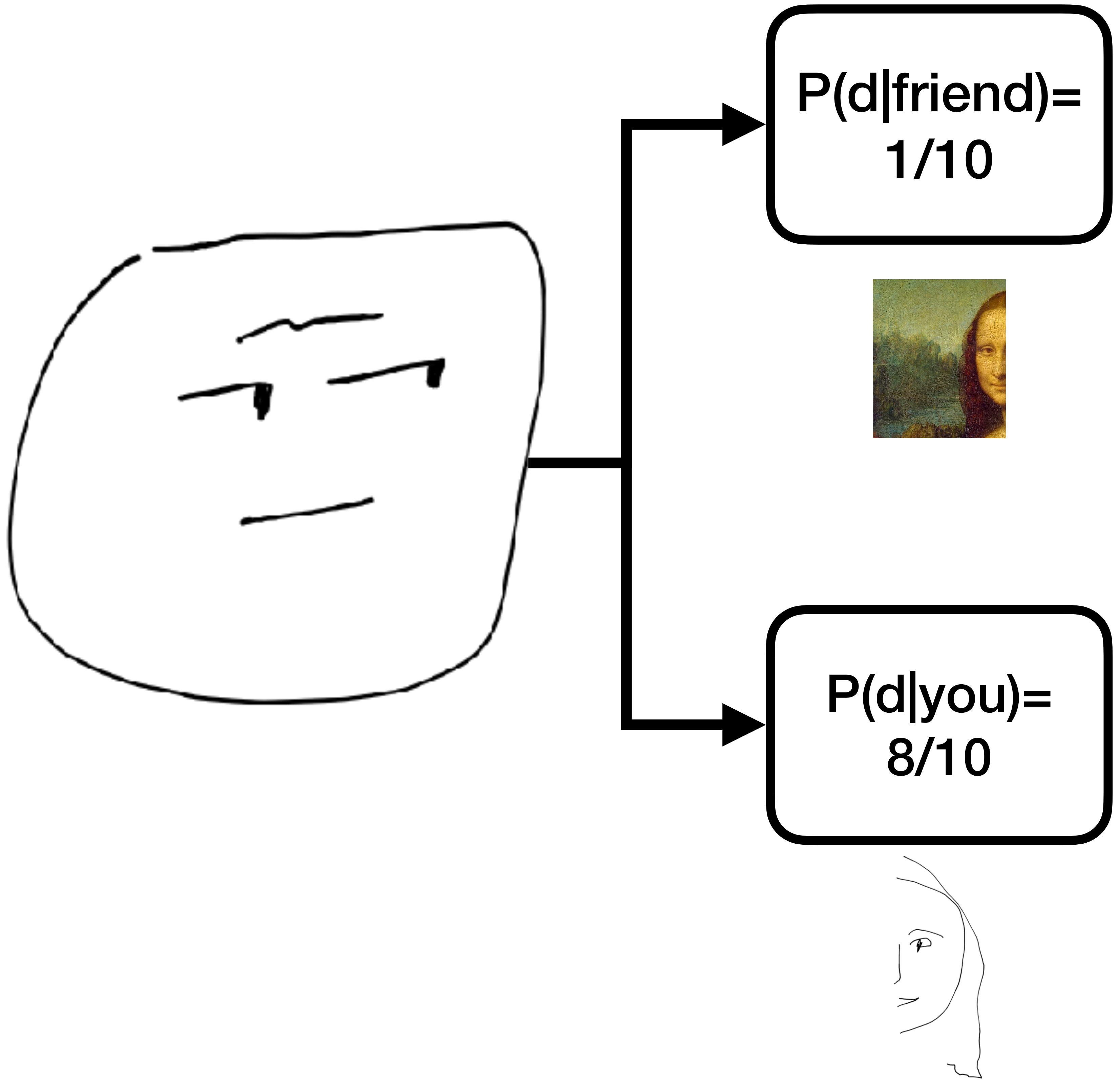

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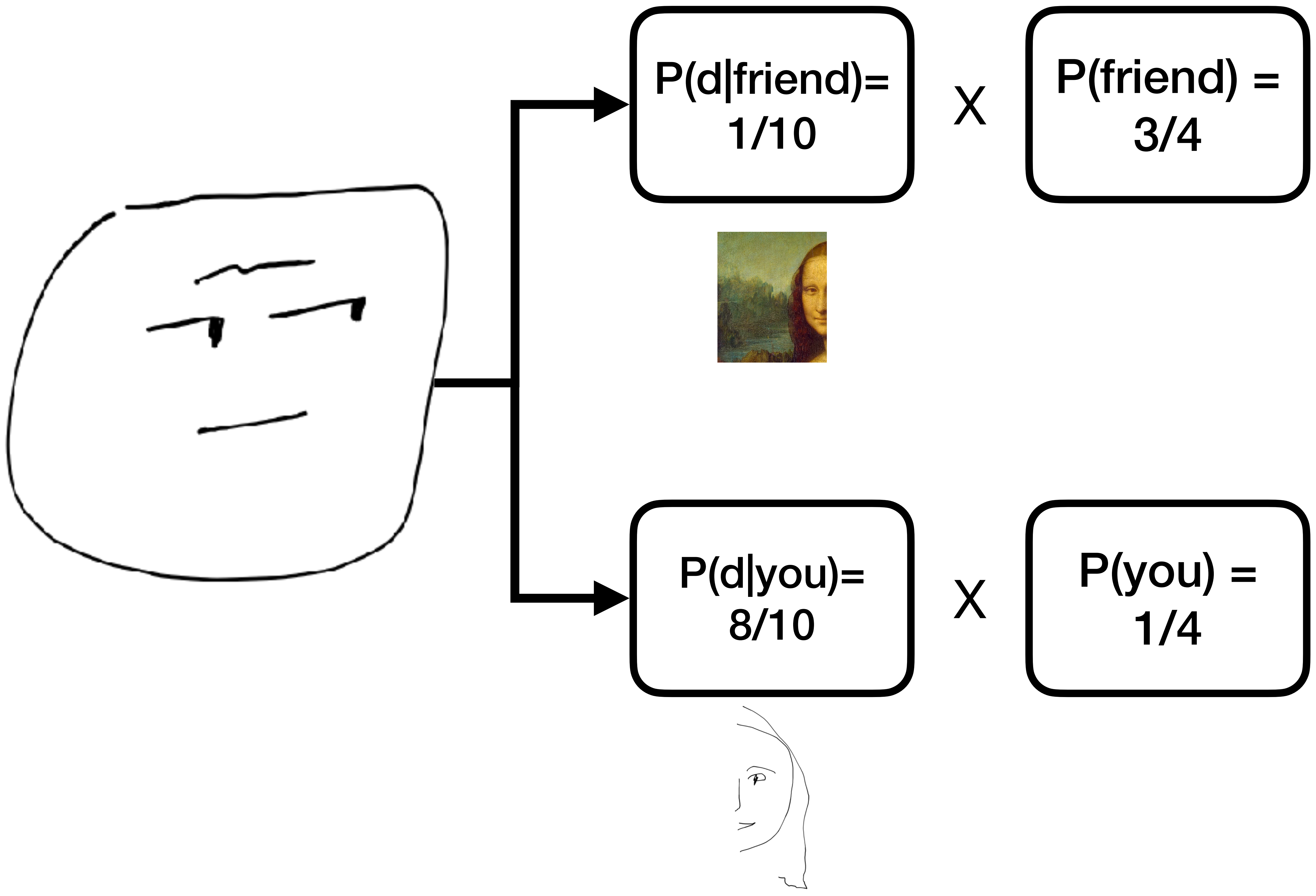

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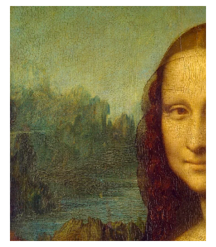
P(d|friend)= 1/10 X P(friend) = 3/4 = 3 / 40



P(d|you)= 8/10 X P(you) = 1/4 = 8 / 40

MAP estimate is you!

Naive Bayes: An "illustration"

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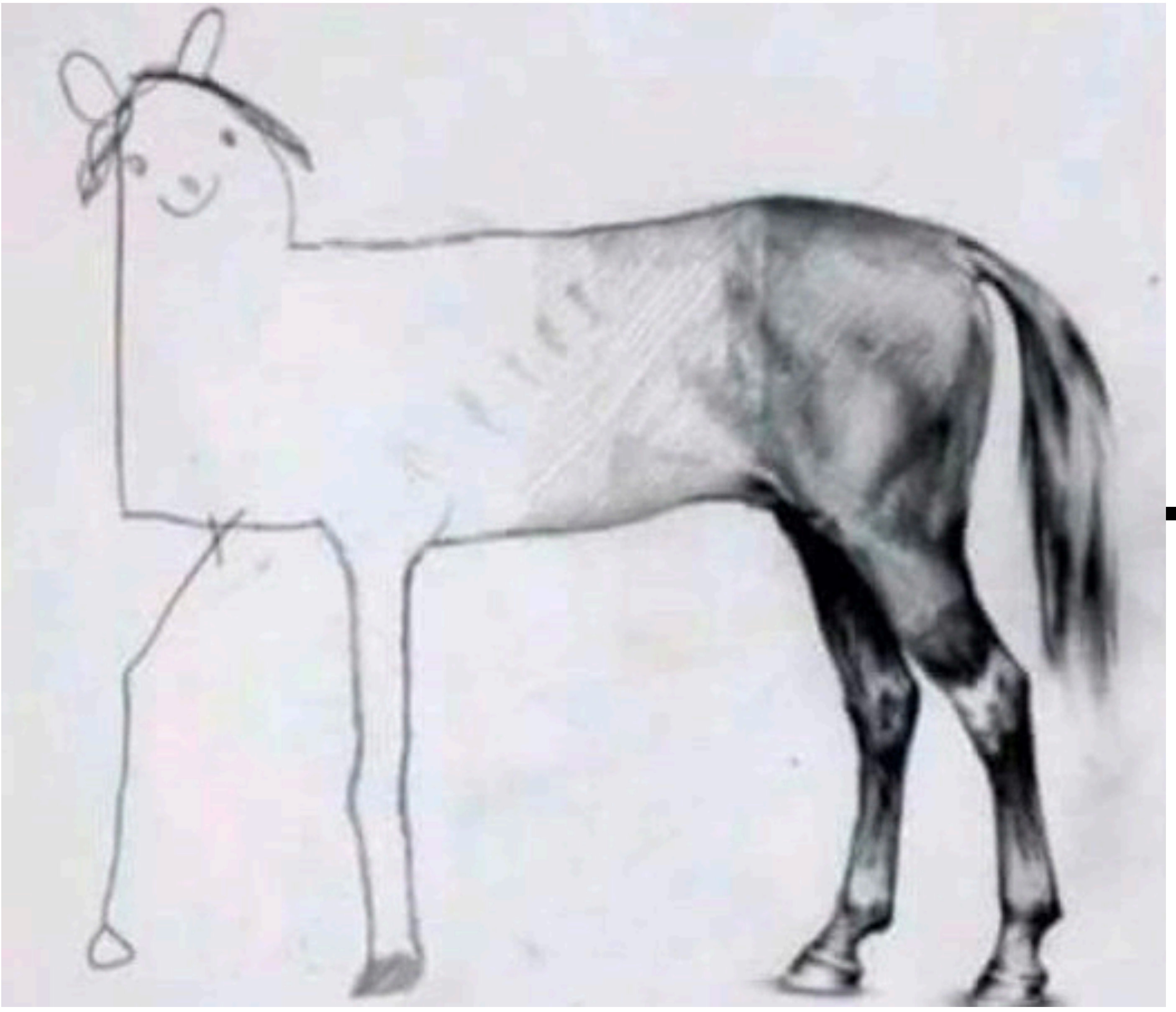
The prior is important when the probabilities are close under each LM!



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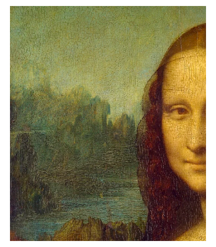

MAP estimate is friend!

P(d|you)= 1/10 X P(you) = 1/4 = 1 / 40

Naive Bayes: One extra detail…

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P(x|you)

$$P(w_1, w_2, \ldots, w_K \mid c) = P(w_1 \mid c) P(w_2 \mid c) \ldots P(w_K \mid c)$$

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$$P(w_1, w_2, \ldots, w_K | c) = P(w_1|c)P(w_2|c)\ldots P(w_K|c)$$

To simplify our LM, we use unigrams. This is equivalent to saying, we assume all words are **independent** of each other. This is the "naive" assumption of Naive Bayes

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2. We don't know $P(c \mid d)$, but we know how to estimate $P(d \mid c)$ using a simple LM! → we can get $P(c \mid d)$ using Bayes' rule!
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4. To estimate $P(d \mid c)$ let's be lazy and choose the simplest possible LM that assume (Naively) that each word is independent - the unigram
5. Combine 3 + 4 and you can find the MAP estimate: $c_{MAP} = \arg\max_{c \in C} P(d \mid c)P(c)$

Advantages of Naive Bayes

- Very fast, low storage requirements
- Robust to irrelevant features

  Irrelevant features cancel each other without affecting results

- Optimal if the independence assumptions hold

  If assumed independence is correct, this is the 'Bayes optimal' classifier

- A good dependable baseline for text classification

  However, other classifiers can give better accuracy

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Compared to NB, with LR we take a more direct approach: directly compute $P(c \mid d)$ given a set of features constructed from the input document d.

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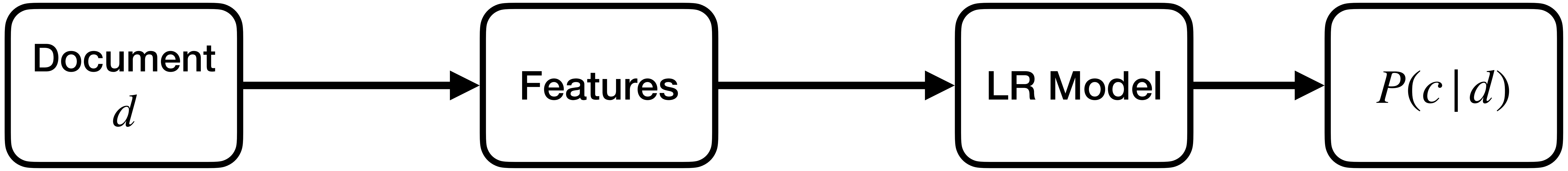

Logistic Regression: Features

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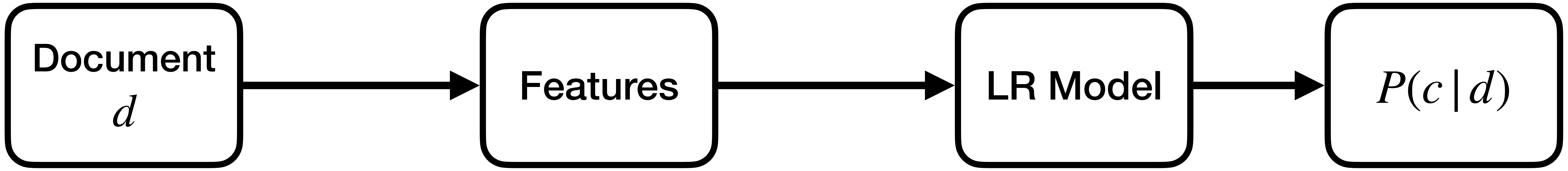

Var	Definition	Value
x_1	count(positive lexicon) $\in$ doc)	3
x_2	count(negative lexicon) $\in$ doc)	2
x_3	$\begin{cases} 1 & \text{if "no"} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	1
x_4	count(1st and 2nd pronouns $\in$ doc)	3
x_5	$\begin{cases} 1 & \text{if "!"} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	0
x_6	log(word count of doc)	$\ln(64) = 4.15$

This is the feature vector x for some input document d

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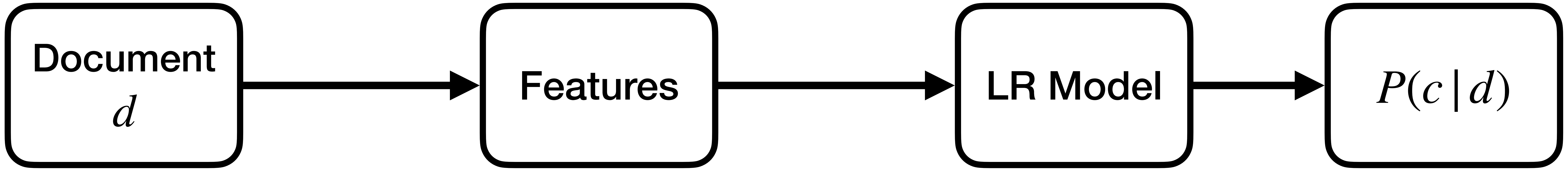

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The features to use is a design decision. A natural default is to use a vector $x \in \mathbb{R}^{|V|}$ where each dim is the counts of one word in the vocabulary. (BOW)

Logistic Regression: LR Model

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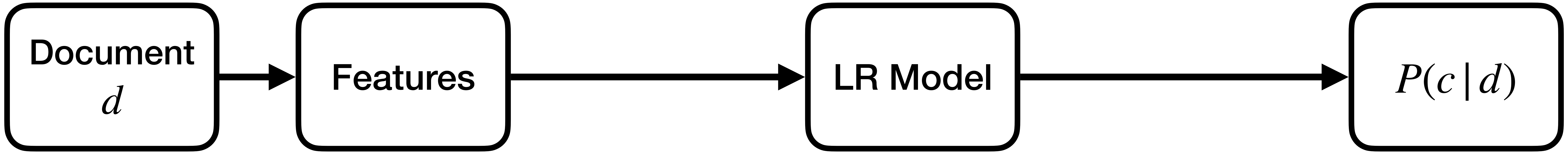
Now given some feature vector x how do we turn this to a probability?



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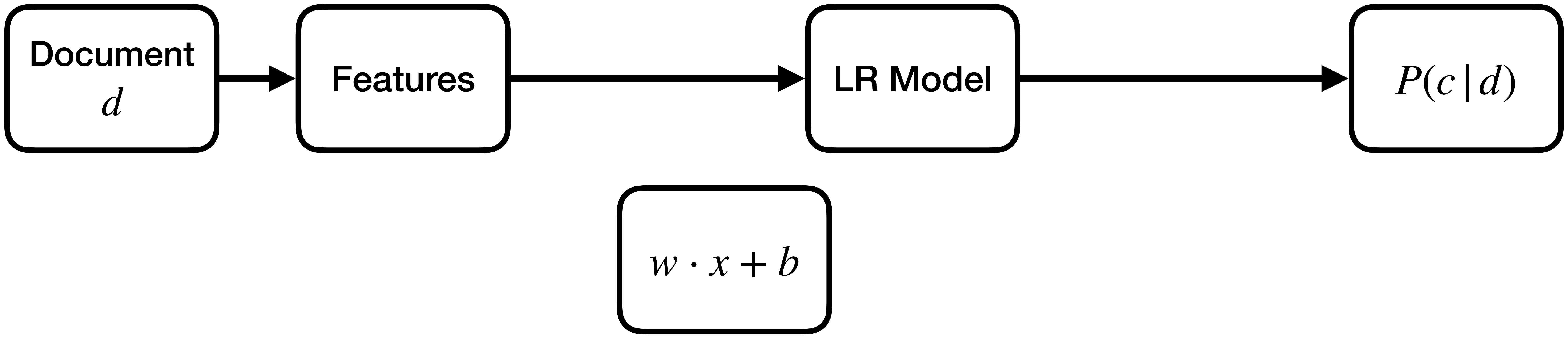
1. Convert the features to a number. The higher the number, the more confident we are that the document belongs to a class. We call these numbers **logits.**



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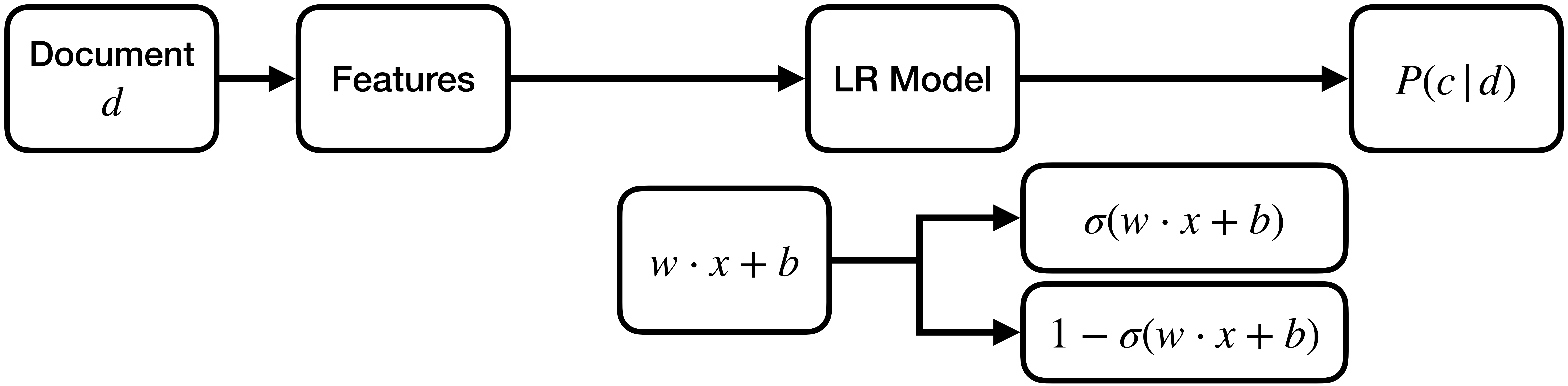
1. Convert the features to a number. The higher the number, the more confident we are that the document belongs to a class. We call these numbers **logits.**
2. Normalize the logits using sigmoid so we get a well-defined probability distribution.
    1. For more than 2 classes we use the softmax, which is the m > 2 generalization of sigmoid



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How to set $\theta = (w, b)$? Use the MLE principle! Set θ such that $P(\mathscr{D})$ is maximized. This is analogous to setting the n-gram probabilities such that the probability of the train corpus is maximal.

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The j-th value of the feature vector $\mathbf{x}_i$

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    1. We can use a BOW, where each dim in $x \in \mathbb{R}^{|V|}$ is the # of times a word in V appears
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6. We can then use GD to minimize the CE loss! Since the function is convex, we will converge to the optimum.

Logistic Regression: what's good and what's not

- More freedom in designing features
  - No strong independence assumptions like Naive Bayes
  - Can even have the same feature twice! (*why?*)
- May not work well on small datasets (compared to Naive Bayes)
- Interpreting learned weights can be challenging

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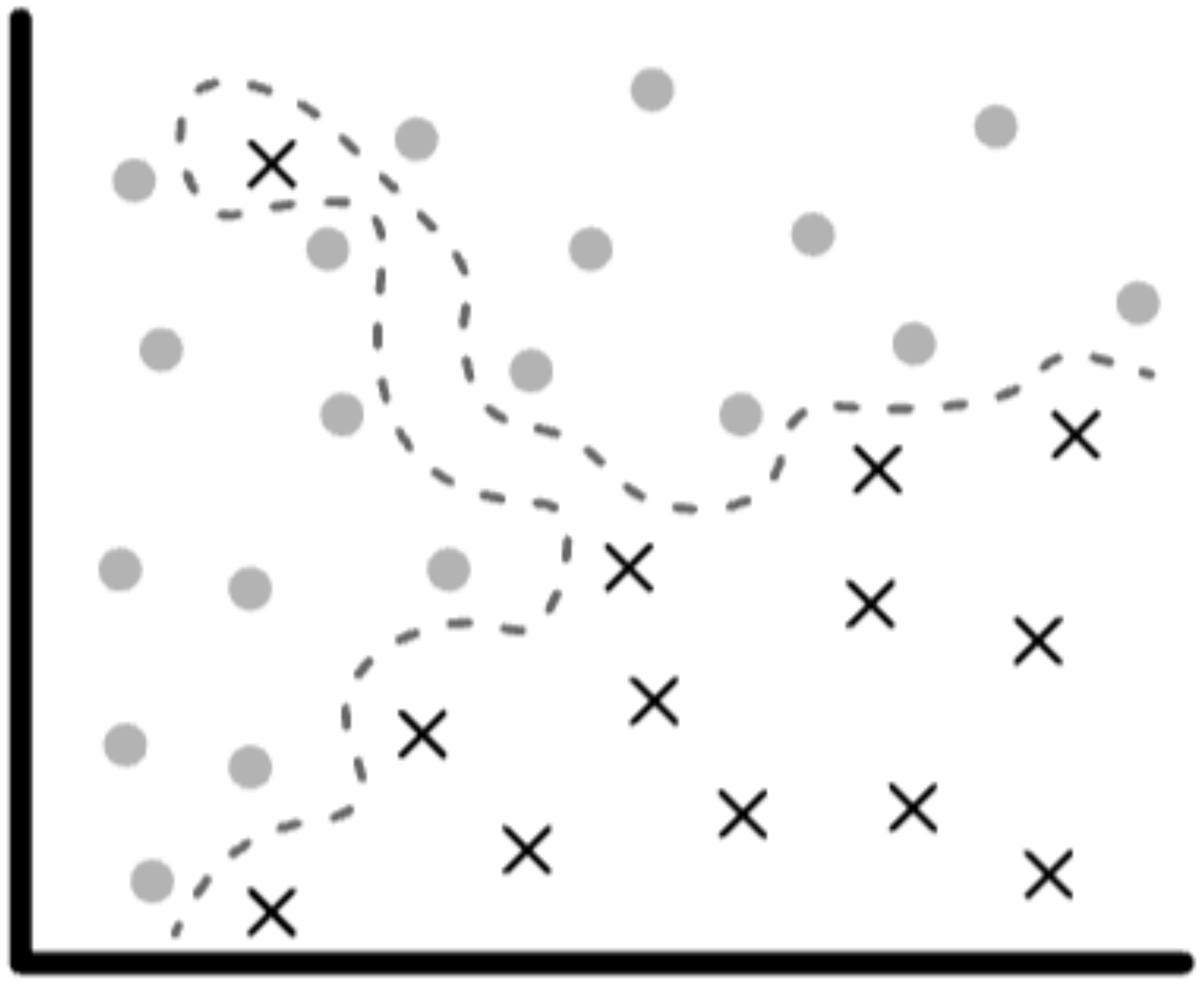

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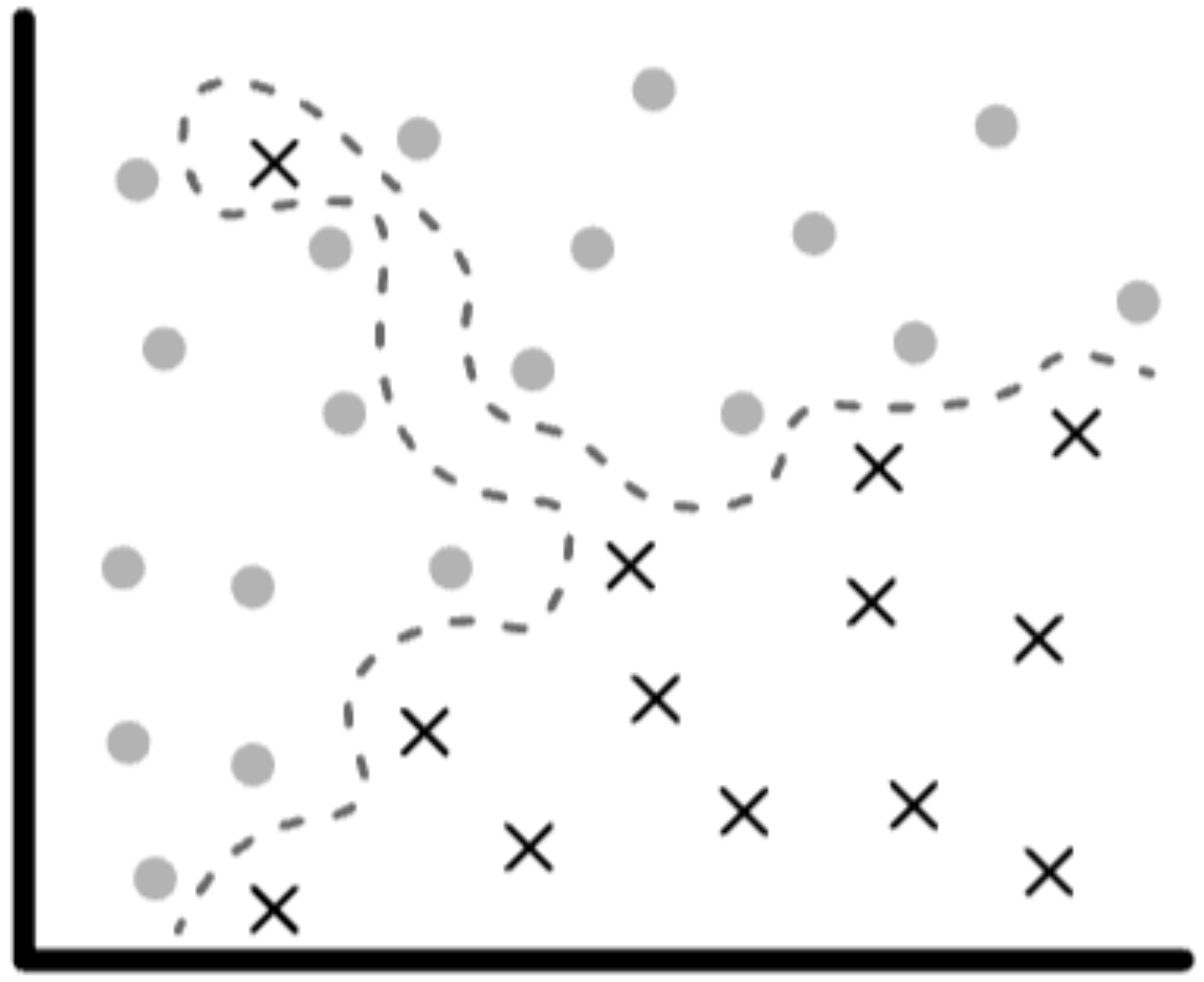

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- The key difference between ridge and lasso regression:
    - Weights in Lasso go to 0, so it can be used for feature selection!