1. Intro to Colab & Language Models

Austin W.

Slides based on those of Jens T., Ameet D., Chris S., and everyone else they based theirs on

Logistics

- Precepts are Fridays, 1 - 2pm ET, COS 402 (for now)
- Office Hours (all be in Friend 003):
  - Alex: Thurs 7 - 8pm
  - Austin: Right after precept
  - Howard: Mon 11-12pm
  - Samyak: Friday 12-1pm
- All assignments should be done on Colab! To maximize OH efficiency we will not be debugging problems with incompatible local Jupyter instances.

Today’s Topics

1. Google Colab walkthrough
2. Lecture review: language models

Google Colab Demo

Useful Resources

- Working with Colab
- Working with LaTeX
- Submitting Assignments
- Feel free to post any issues with any of these on Ed!

Language Models Review

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Definition: A **language model** is a probabilistic model over sequences of words (tokens).

More explicitly, a probability over a sequence is the joint probability of the tokens $P(w_1, w_2, \ldots, w_n)$

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Given an (ideally very large) sequence of words (called a corpus) how do we set $P(w_n \,|\, w_1, \ldots, w_{n-1})$?

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Why not let $P(w_n \mid w_1, \ldots, w_{n-1})$ = 1 for w_n with the max count from the corpus, 0 for all other words and then apply smoothing?

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Why not let $P(w_n \mid w_1, \ldots, w_{n-1})$ = 1 for w_n with the max count from the corpus, 0 for all other words and then apply smoothing?

MLE Principle: We want to set $P(w_n \mid w_1, \ldots, w_{n-1})$ such that the probability of the corpus is maximized! → perplexity is minimized

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This is the **provable way to set the probabilities so corpus perplexity is minimized:**

$$P(w_3 \mid w_1, w_2) \leftarrow \frac{\mathrm{Count}(w_1, w_2, w_3)}{\mathrm{Count}(w_1, w_2)}$$

where $\mathrm{Count}(w_1, w_2, w_3)$ is the number of times the sequence "$w_1 w_2 w_3$" occurs in the corpus.

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How to evaluate a language model?

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How to evaluate a language model? For a test corpus S with n words $w_1, w_2, \ldots, w_n$

$$\text{ppl}(S) = P(w_1, . . . , w_n)^{-1/n} = \exp\left(-\frac{1}{n}\sum_{i=1}^{n} \log P(w_i \mid w_1, \ldots, w_{i-1})\right)$$

where n is the total number of words in the corpus

Lower perplexity means the model accurately describes the corpus. Intuitively, you can think of perplexity as the **average branching factor** (i.e. between how many words is the model choosing when predicting the next word).

Language Models Review

Intuition on perplexity

If our k-gram model (with vocabulary V) has following probability:

$$P(w \mid w_{i-k}, \ldots w_{i-1}) = \frac{1}{|V|} \quad \forall w \in V$$

$$\text{ppl}(S) = e^{x} \quad \text{where}$$
$$x = -\frac{1}{n}\sum_{i=1}^{n} \log P(w_i \mid w_1 \ldots w_{i-1})$$

what is the perplexity of the test corpus?

A) $e^{|V|}$ B) $|V|$ C) $|V|^2$ D) $e^{-|V|}$

$$\text{ppl} = e^{-\frac{1}{n} n \log(1/|V|)} = |V|$$

Measure of model's uncertainty about next word (aka `average branching factor')

branching factor = # of possible words following any word

Language Models Review

Calculating the probabilities exactly for every sequence is **infeasible** because of the sheer number of possible sequences $(|V|^n)$

- Impossible for training corpus to have counts for every conceivable $\mathrm{Count}(w_1, w_2, \ldots, w_n)$

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Impossible for training corpus to have counts for every conceivable $\text{Count}(w_1, w_2, \ldots, w_n)$

We approximate using the **Markov assumption**:

1st order approximation:

$$P(w_n | w_1, w_2, \ldots, w_{n-1}) \approx P(w_n | w_{n-1})$$

2nd order approximation:

$$P(w_n | w_1, w_2, \ldots, w_{n-1}) \approx P(w_n | w_{n-2}, w_{n-1})$$

kth order approximation:

$$P(w_n | w_1, w_2, \ldots, w_{n-1}) \approx P(w_n | w_{n-k}, \ldots, w_{n-2}, w_{n-1})$$

Language Models Review

An **n-gram language model** is an $(n-1)^{\mathrm{th}}$ order Markov approximation:

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Unigram (1 - gram) model:

$$P(w_1, w_2, \ldots, w_n) \approx P(w_1)P(w_2)\ldots P(w_n) = \prod_{i=1}^{n} P(w_i)$$

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Bigram (2-gram) model:

$$P(w_1, w_2, \ldots, w_n) \approx P(w_1)P(w_2 \mid w_1)\ldots P(w_n \mid w_{n-1}) = \prod_{i=1}^{n} P(w_i \mid w_{i-1})$$

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N-gram model:

$$P(w_1, w_2, \ldots, w_n) \approx \prod_{i=1}^{n} P(w_i \mid w_{i-n+1}, \ldots, w_{i-2}, w_{i-1})$$

Language Models Review

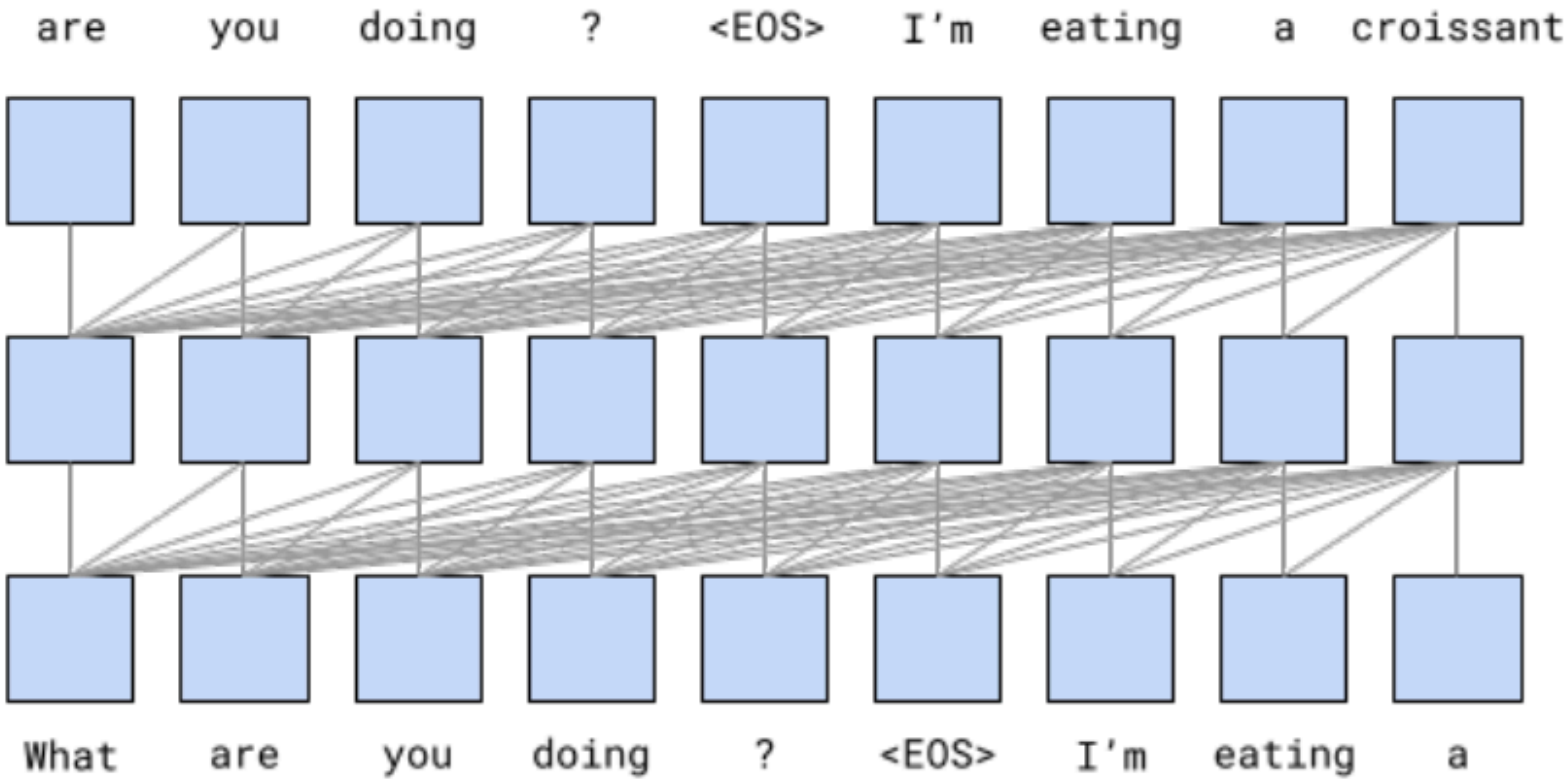
Generating from a language model

- Given a language model, how to generate a sequence?

Trigram $$P(w_1, w_2, \ldots, w_n) = \prod_{i=1}^{n} P(w_i \mid w_{i-2}, w_{i-1})$$

- Generate the first word $w_1 \sim P(w)$
- Generate the second word $w_2 \sim P(w \mid w_1)$
- Generate the third word $w_3 \sim P(w \mid w_1, w_2)$
- Generate the fourth word $w_4 \sim P(w \mid w_2, w_3)$
- …

Left to Right Generation Surprisingly Powerful…



Thoppilan et al. 2022 LaMDA Langague Models for Dialogue Applications

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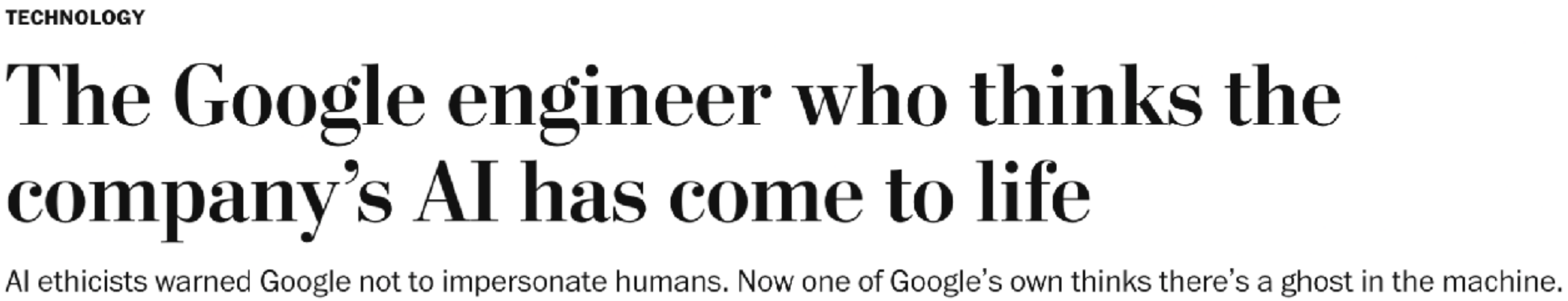
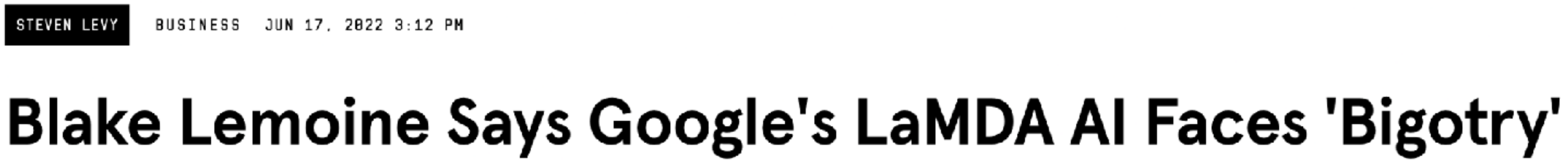


TECHNOLOGY

The Google engineer who thinks the company's AI has come to life

AI ethicists warned Google not to impersonate humans. Now one of Google's own thinks there's a ghost in the machine.



STEVEN LEVY BUSINESS JUN 17, 2022 3:12 PM

Blake Lemoine Says Google's LaMDA AI Faces 'Bigotry'

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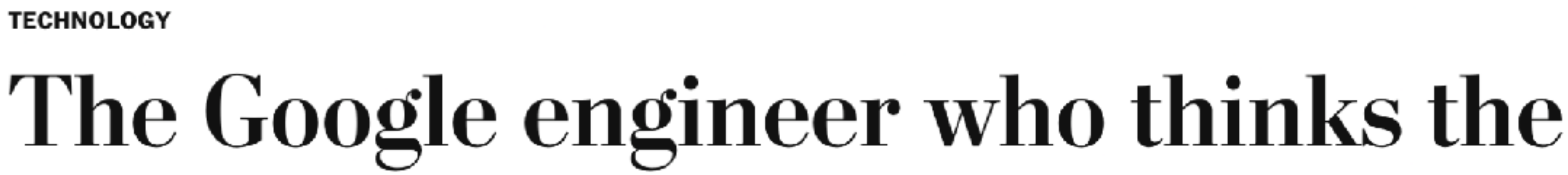

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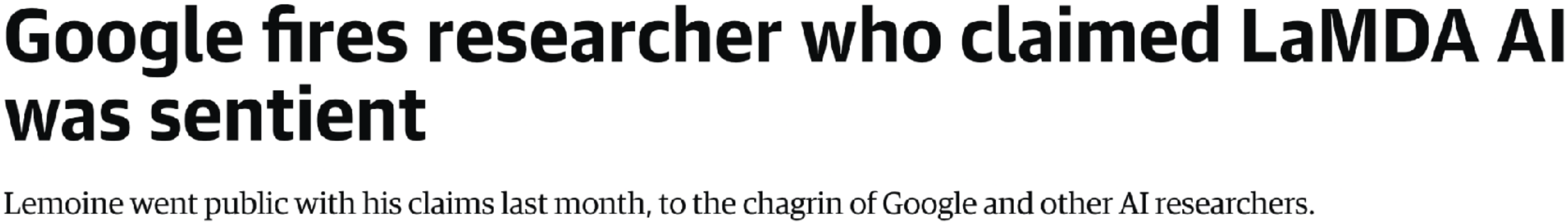
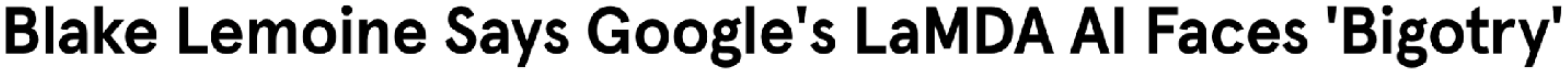


Google fires researcher who claimed LaMDA AI was sentient

Lemoine went public with his claims last month, to the chagrin of Google and other AI researchers.



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To make estimating these probabilities tractable, we use **Markov assumption** (e.g. bigram)

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We set these conditional probabilities to **minimize the perplexity of training corpus.** For trigram:

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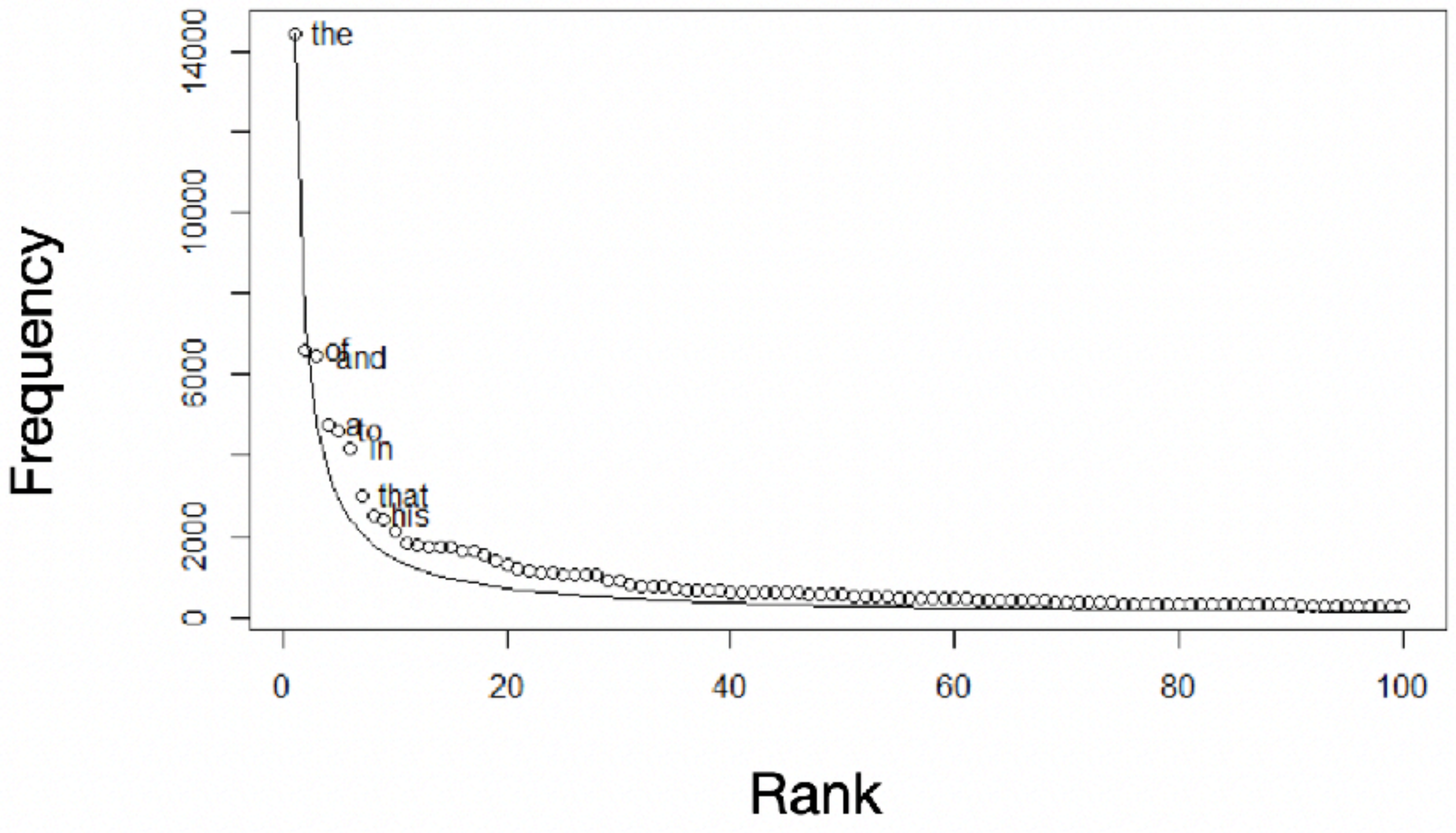
We evaluate using **perplexity:**

$$\text{ppl}(S) = P(w_1, \ldots, w_n)^{-1/n} = \exp\Big(-\frac{1}{n} \sum_{i=1}^{n} \log P(w_i \mid w_1, \ldots, w_{i-1})\Big)$$

Smoothing

We want our models to accurately describe our languages. But, languages have a **long tail** and we have **finite data → Not all n-grams will be observed in the training data!**



$$freq \propto \frac{1}{rank}$$

Zipf's Law

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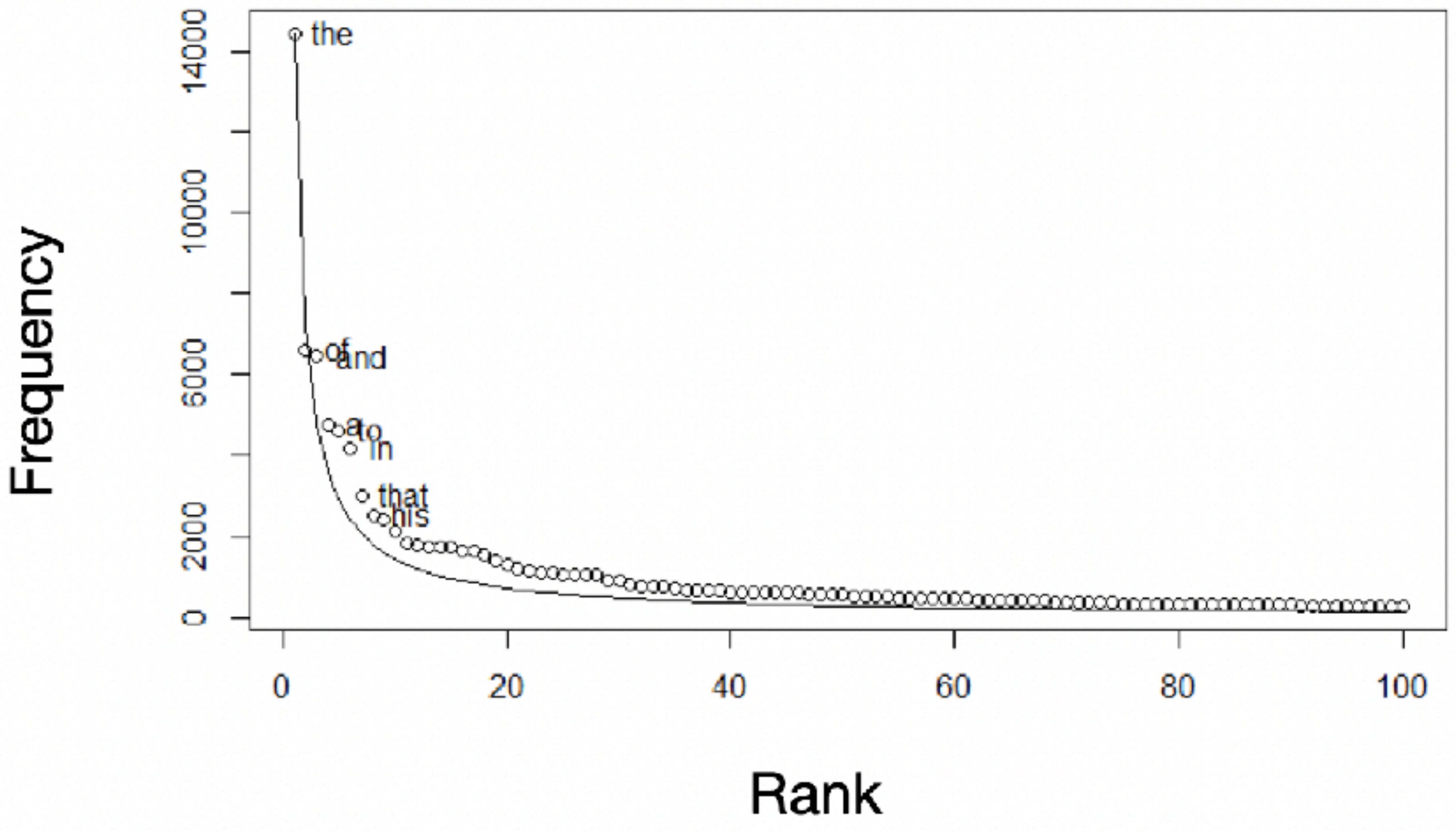
We want our models to accurately describe our languages. But, languages have a **long tail** and we have **finite data → Not all n-grams will be observed in the training data!**

How can we help our models compensate for this sparsity? **Smoothing!**

- Additive
- Discounting
- Back-off
- Interpolation



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Zipf's Law

Smoothing

Additive smoothing (Laplace): add a small count to each n-gram

- Simplest form of smoothing: Just add α to all counts and renormalize!

- Max likelihood estimate for bigrams:

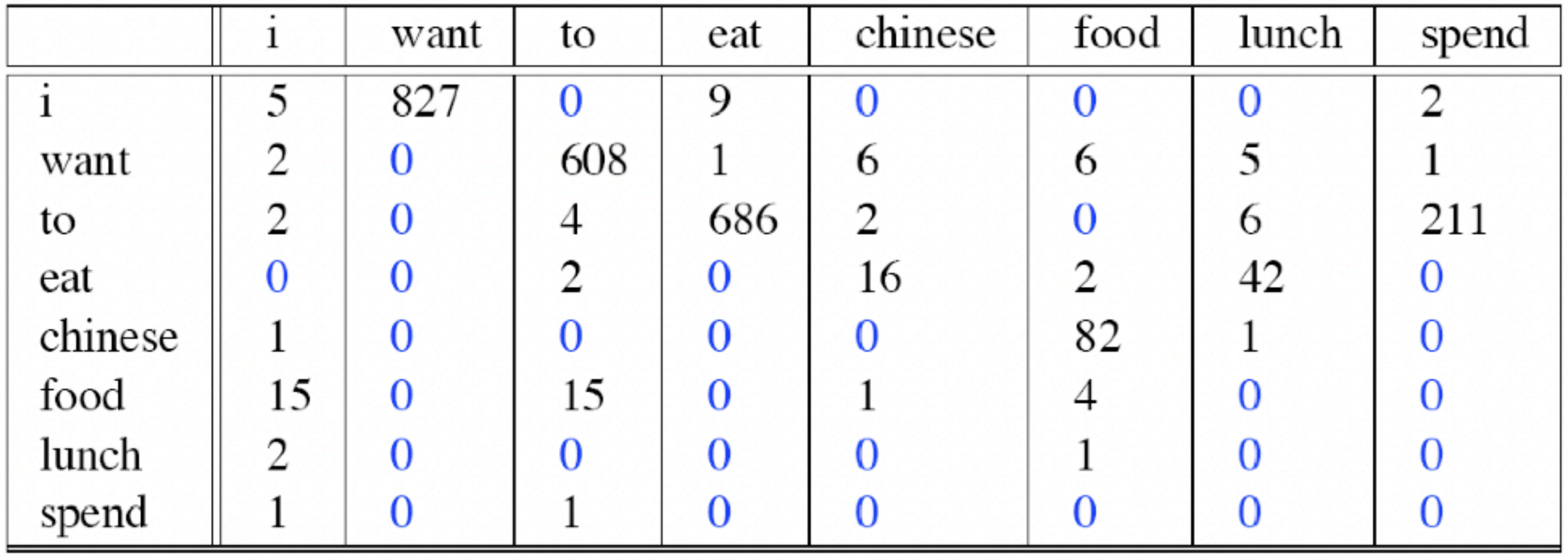
$$P(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i)}{C(w_{i-1})}$$

- After smoothing:

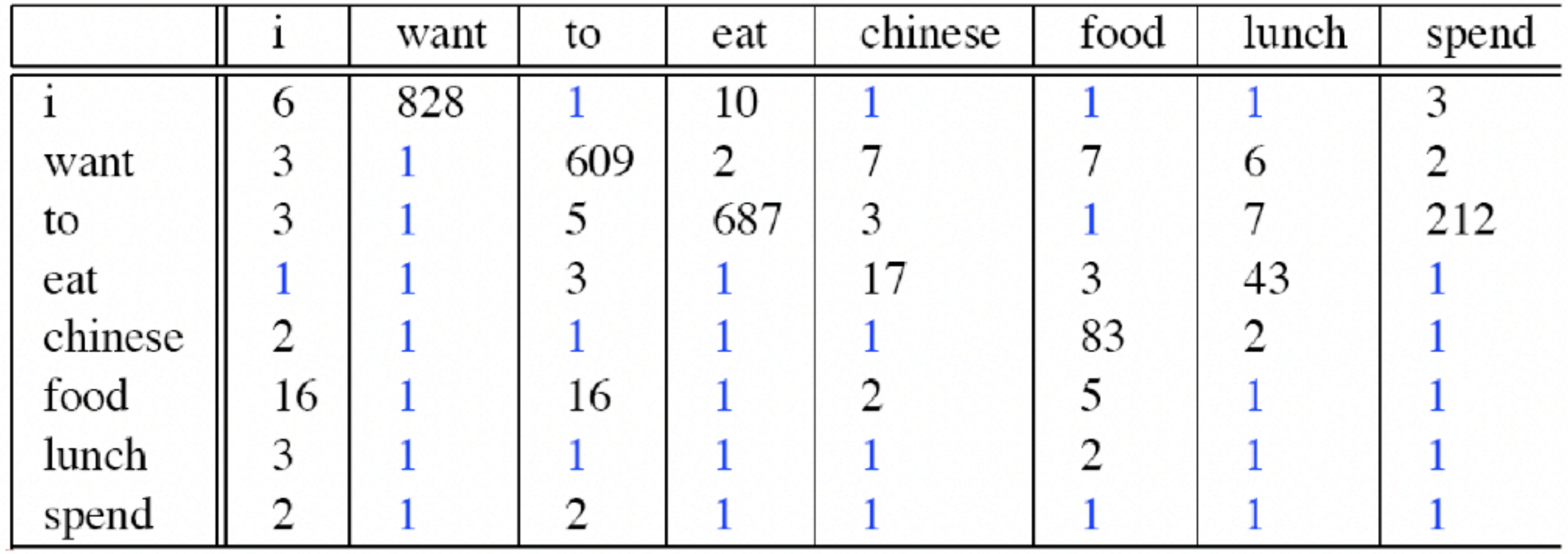
$$P(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i) \boxed{+ \alpha}}{C(w_{i-1}) \boxed{+ \alpha|V|}}$$

Smoothing

Additive smoothing (Laplace): add a small count (α) to each n-gram



	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0



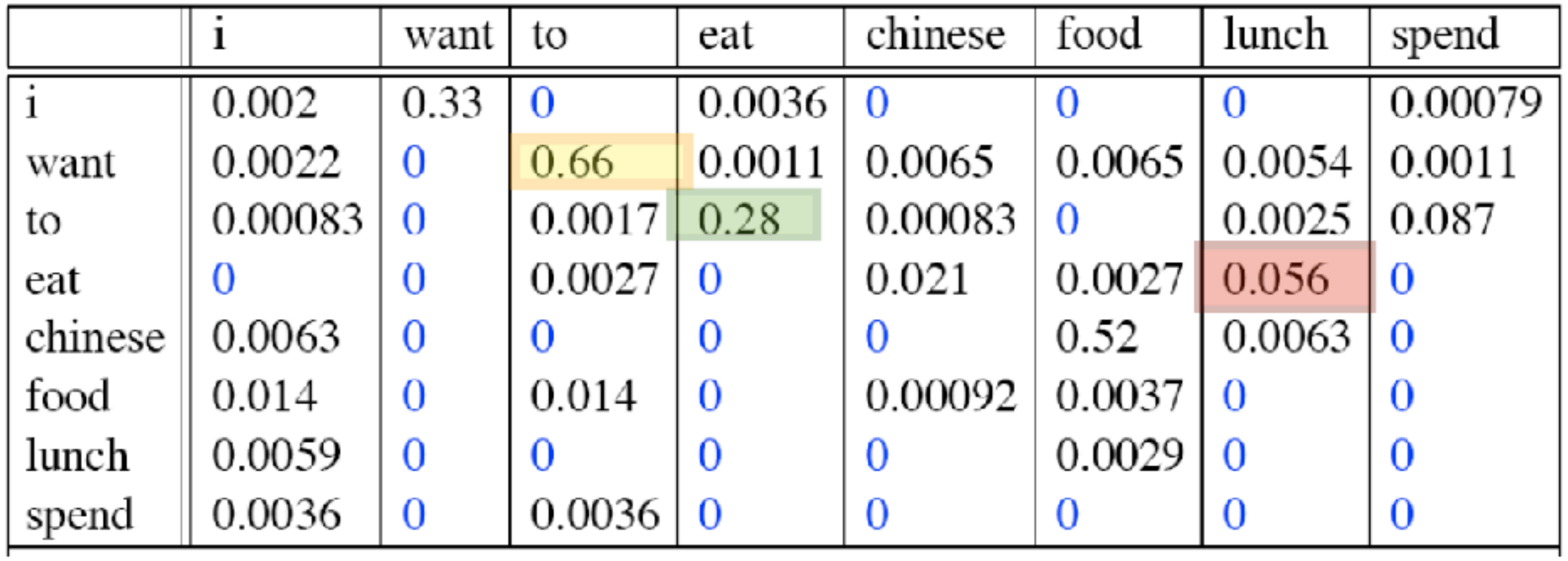
	i	want	to	eat	chinese	food	lunch	spend
i	6	828	1	10	1	1	1	3
want	3	1	609	2	7	7	6	2
to	3	1	5	687	3	1	7	212
eat	1	1	3	1	17	3	43	1
chinese	2	1	1	1	1	83	2	1
food	16	1	16	1	2	5	1	1
lunch	3	1	1	1	1	2	1	1
spend	2	1	2	1	1	1	1	1

Add 1 (α = 1) observation to each bigram

Smoothing

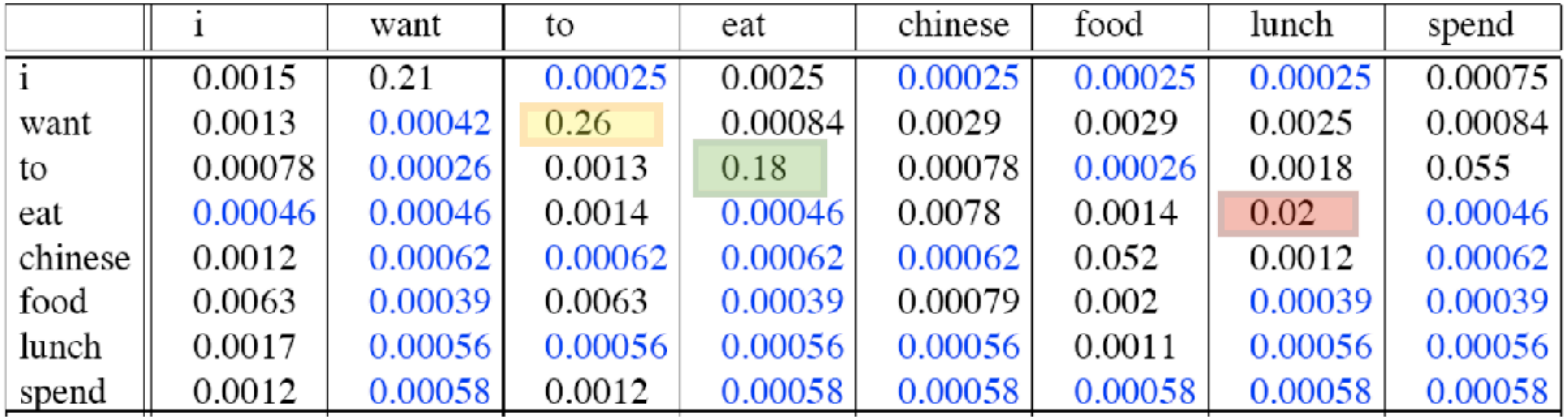
Additive smoothing (Laplace): add a small count (α) to each n-gram

Original:



	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Smoothed:



	i	want	to	eat	chinese	food	lunch	spend
i	0.0015	0.21	0.00025	0.0025	0.00025	0.00025	0.00025	0.00075
want	0.0013	0.00042	0.26	0.00084	0.0029	0.0029	0.0025	0.00084
to	0.00078	0.00026	0.0013	0.18	0.00078	0.00026	0.0018	0.055
eat	0.00046	0.00046	0.0014	0.00046	0.0078	0.0014	0.02	0.00046
chinese	0.0012	0.00062	0.00062	0.00062	0.00062	0.052	0.0012	0.00062
food	0.0063	0.00039	0.0063	0.00039	0.00079	0.002	0.00039	0.00039
lunch	0.0017	0.00056	0.00056	0.00056	0.00056	0.0011	0.00056	0.00056
spend	0.0012	0.00058	0.0012	0.00058	0.00058	0.00058	0.00058	0.00058

Smoothing

Additive smoothing (Laplace): add a small count (α) to each n-gram

As α increases, we approach the uniform distribution.
Add α often removes too much probability mass / too simple to work well in practice

$$P(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i) \boxed{+ \alpha}}{C(w_{i-1}) \boxed{+ \alpha|V|}}$$

Smoothing

Discounting: Take probability mass from each of the observed n-grams. Redistribute it among unseen n-grams.

$$P(w_i \mid w_{i-1}) = \begin{cases} \dfrac{\text{Count}(w_{i-1}, w_i) - d}{\text{Count}(w_{i-1})} & \text{Count}(w_{i-1}, w_i) > 0 \\ \alpha(w_{i-1}) \cdot \dfrac{P(w_i)}{\sum_{w:\text{Count}(w_{i-1},w)=0} P(w)} & \text{Count}(w_{i-1}, w_i) = 0 \end{cases}$$

Left-over probability mass to be redistributed (either uniformly or according to unigram probabilities as above)

Smoothing

Discounting: Take probability mass from each of the observed n-grams. Redistribute it among unseen n-grams.

$$P(w_i \mid the) = \begin{cases} \dfrac{\text{Count}(the, w_i) - d}{\text{Count}(the)} & \text{Count}(the, w_i) > 0 \\ \alpha(the) \cdot \dfrac{P(w_i)}{\sum_{w:\text{Count}(the,w)=0} P(w)} & \text{Count}(the, w_i) = 0 \end{cases}$$

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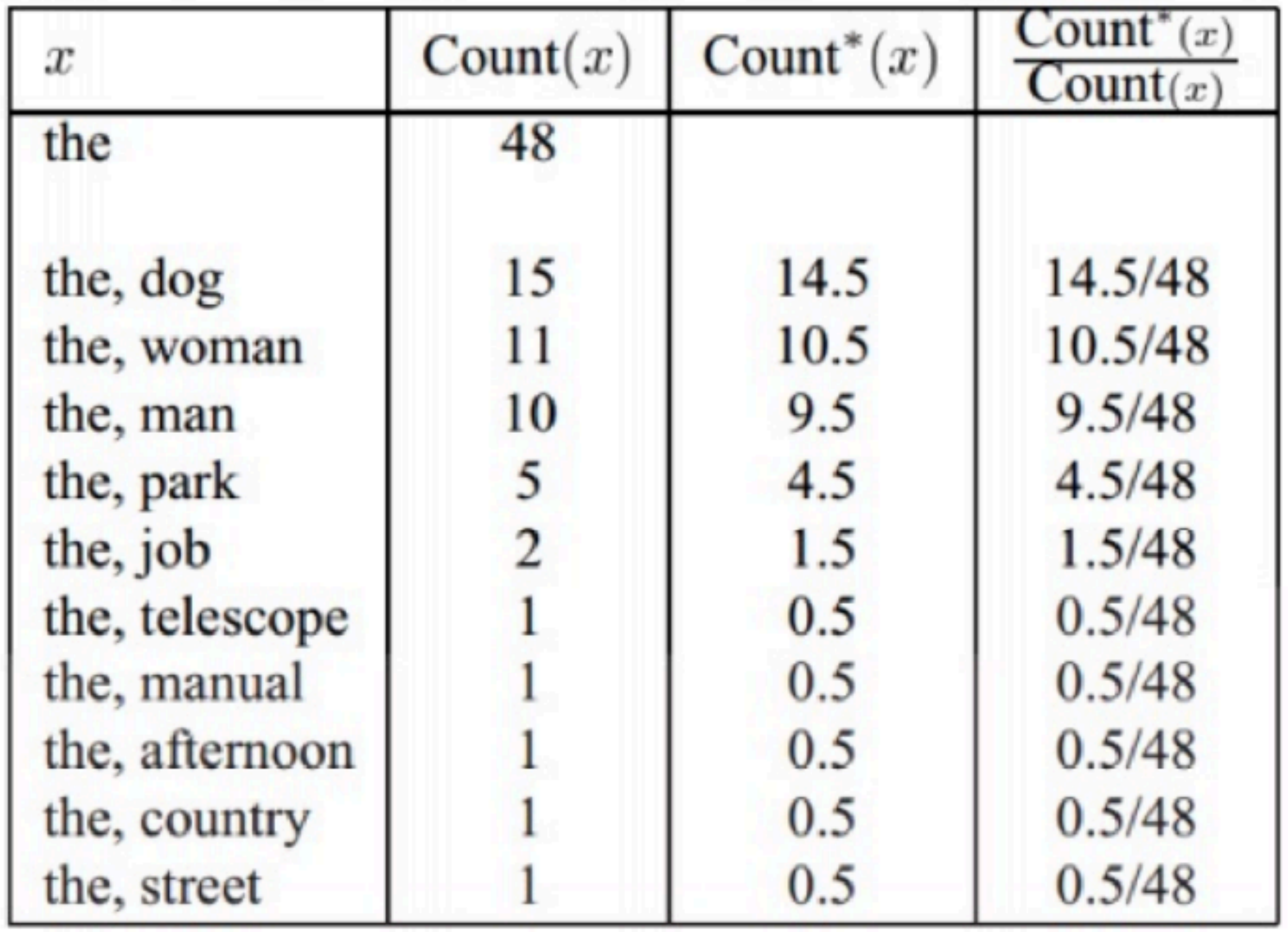
- Define Count*(x) = Count(x) - 0.5

- Missing probability mass:

$$\alpha(w_{i-1}) = 1 - \sum_{w} \frac{\text{Count}^*(w_{i-1,w})}{\text{Count}(w_{i-1})}$$

$$\alpha(\text{the}) = 10 \times 0.5/48 = 5/48$$

- Divide this mass between words *w* for which Count(the, *w*) = 0



x	$\text{Count}(x)$	$\text{Count}^*(x)$	$\frac{\text{Count}^*(x)}{\text{Count}(x)}$
the	48		
the, dog	15	14.5	14.5/48
the, woman	11	10.5	10.5/48
the, man	10	9.5	9.5/48
the, park	5	4.5	4.5/48
the, job	2	1.5	1.5/48
the, telescope	1	0.5	0.5/48
the, manual	1	0.5	0.5/48
the, afternoon	1	0.5	0.5/48
the, country	1	0.5	0.5/48
the, street	1	0.5	0.5/48

Smoothing

Discounting: Take probability mass from each of the observed n-grams. Redistribute it among unseen n-grams.

$$P(w_i \mid the) = \begin{cases} \dfrac{\text{Count}(the, w_i) - d}{\text{Count}(the)} & \text{Count}(the, w_i) > 0 \\ \alpha(the) \cdot \dfrac{P(w_i)}{\sum_{w:\text{Count}(the,w)=0} P(w)} & \text{Count}(the, w_i) = 0 \end{cases}$$

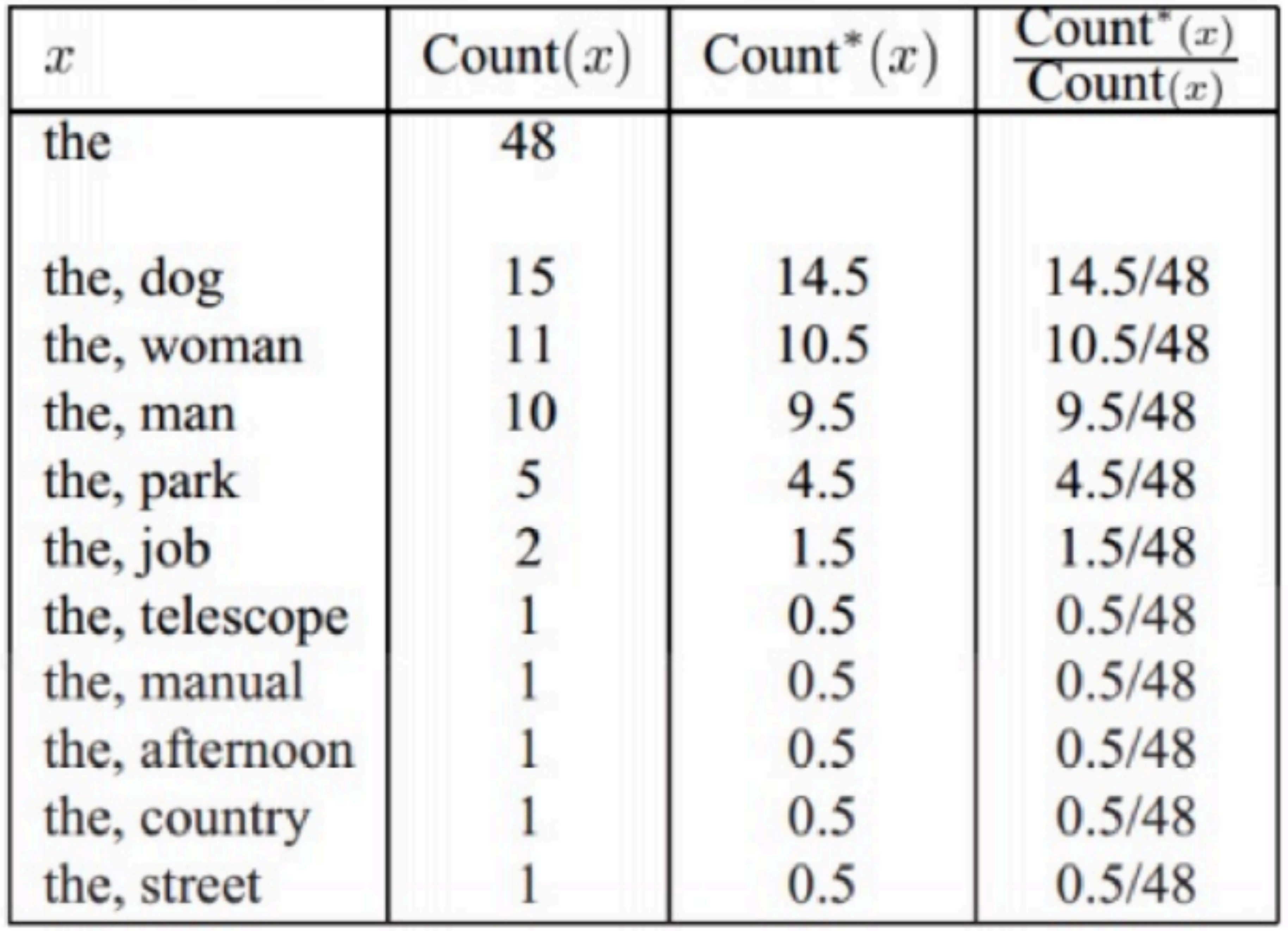
- Define Count*(x) = Count(x) - 0.5

- Missing probability mass:

$$\alpha(w_{i-1}) = 1 - \sum_{w} \frac{\text{Count}^*(w_{i-1,w})}{\text{Count}(w_{i-1})}$$

$$\alpha(\text{the}) = 10 \times 0.5/48 = 5/48$$

- Divide this mass between words *w* for which Count(the, *w*) = 0



x	Count(x)	Count$^*(x)$	$\frac{\text{Count}^*(x)}{\text{Count}(x)}$
the	48		
the, dog	15	14.5	14.5/48
the, woman	11	10.5	10.5/48
the, man	10	9.5	9.5/48
the, park	5	4.5	4.5/48
the, job	2	1.5	1.5/48
the, telescope	1	0.5	0.5/48
the, manual	1	0.5	0.5/48
the, afternoon	1	0.5	0.5/48
the, country	1	0.5	0.5/48
the, street	1	0.5	0.5/48

Counts

the, teacher = 0
the, student = 0

teacher = 1
student = 2

Smoothing

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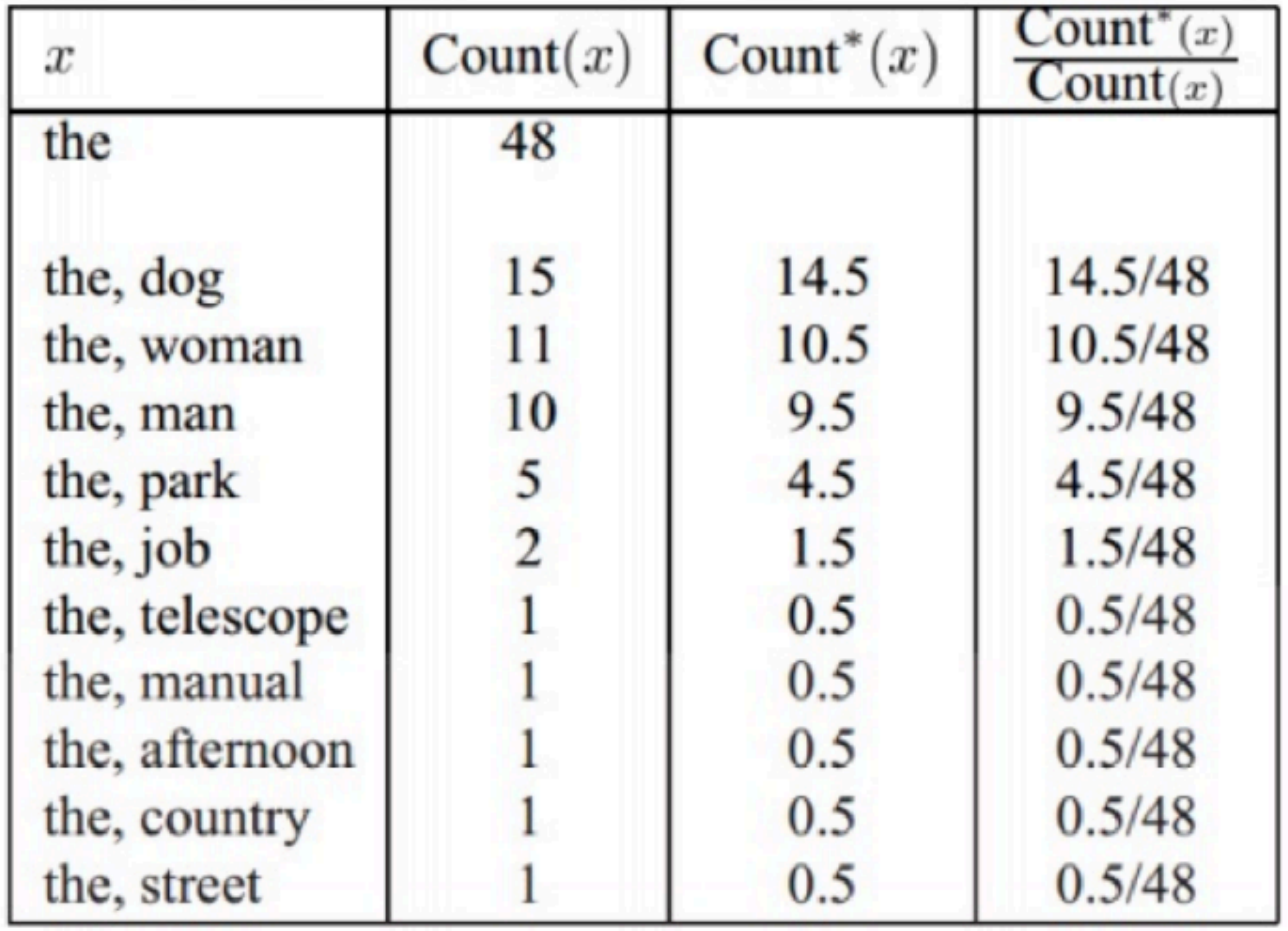

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<u>Counts</u>

the, teacher = 0
the, student = 0

teacher = 1
student = 2

<u>Prob after smoothing</u>

the, teacher = $\frac{5}{48} \times \frac{1}{3}$

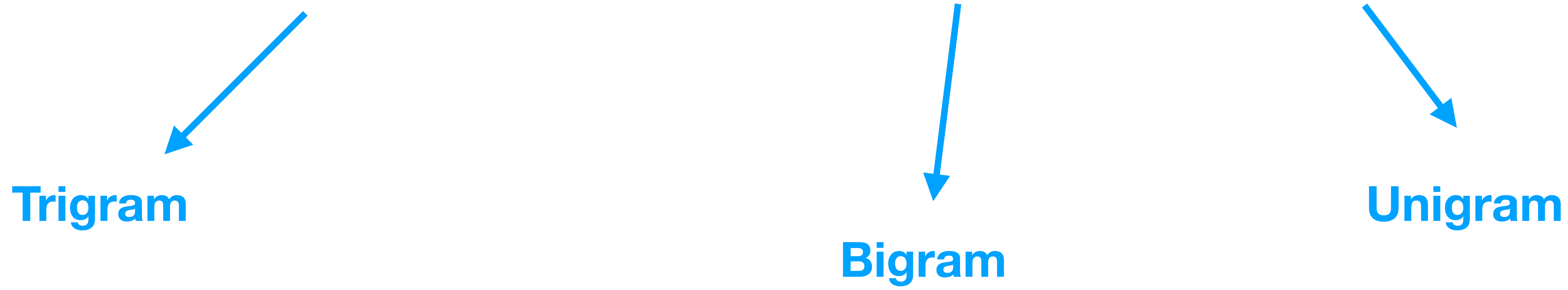
the, student = $\frac{5}{48} \times \frac{2}{3}$

Smoothing

Interpolation: Use a combination of multiple different n-grams.

E.g. Linear interpolation

$$\hat{P}(w_i \mid w_{i-2}, w_{i-1}) = \lambda_1 P(w_i \mid w_{i-2}, w_{i-1}) + \lambda_2 P(w_i \mid w_{i-1}) + \lambda_3 P(w_i) \qquad \sum_i \lambda_i = 1$$



Trigram

Bigram

Unigram

How do we **pick lambdas**? Many ways!

- Use a development set to pick best one
- Average-count (Chen and Goldman, 1996)
- …